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MICHIGAN DEPARTMENT OF ENVIRONMENTAL QUALITY,
SOURCE WATER ASSESSMENT PROGRAM, and the
DETROIT WATER AND SEWERAGE DEPARTMENT

Steady-State Flow Distribution and Monthly Flow Duration in Selected Branches of St. Clair and Detroit Rivers within the Great Lakes Waterway

Water-Resources Investigations Report 01-4135

U.S. Department of the Interior
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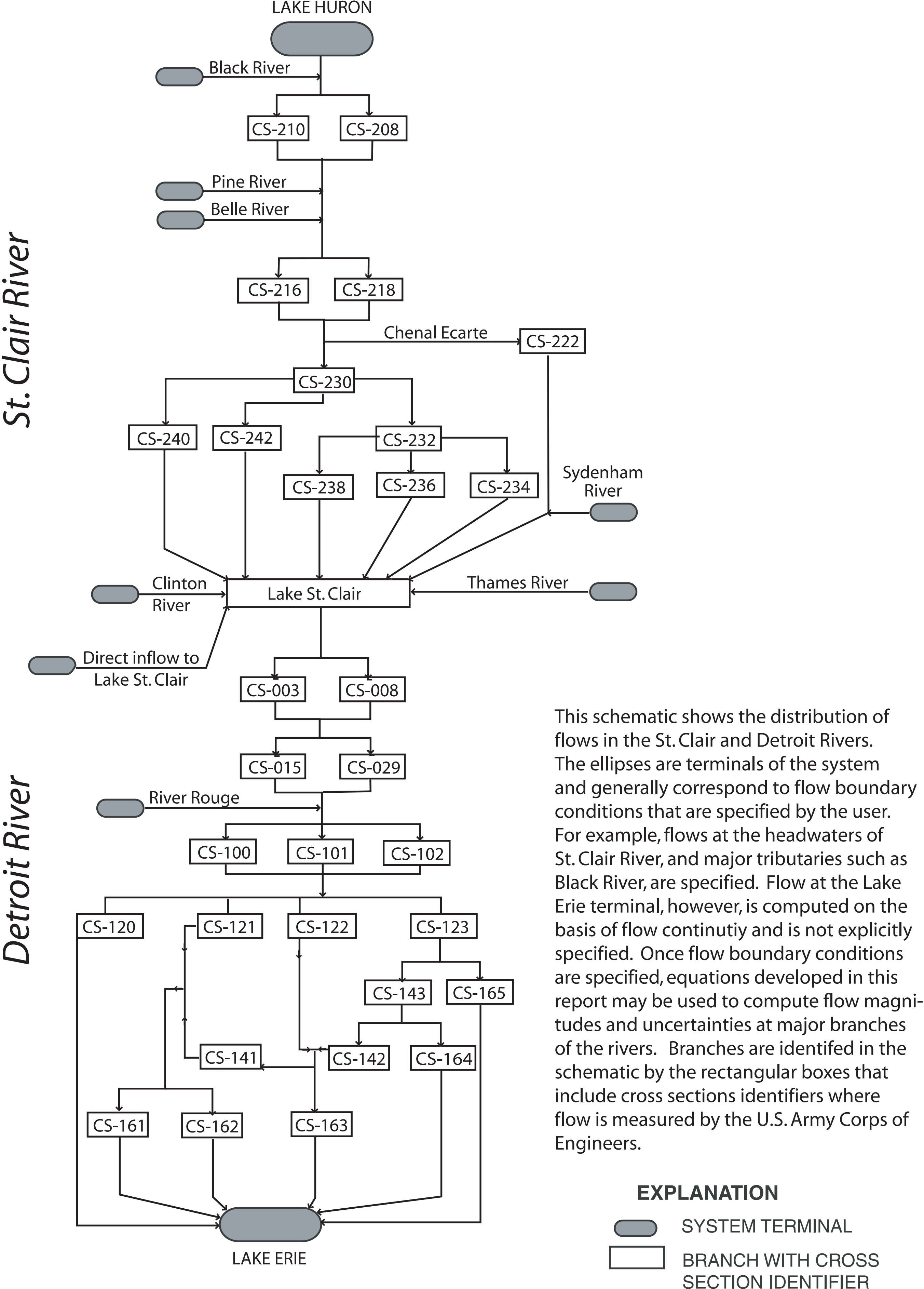
Holtschlag and Koschik—STEADY-STATE FLOW DISTRIBUTION AND MONTHLY FLOW DURATION IN SELECTED BRANCHES OF ST. CLAIR AND DETROIT RIVERS WITHIN THE GREAT LAKES WATERWAY—WRIR 01-4135



This schematic shows the distribution of flows in the St. Clair and Detroit Rivers. The ellipses are terminals of the system and generally correspond to flow boundary conditions that are specified by the user. For example, flows at the headwaters of St. Clair River, and major tributaries such as Black River, are specified. Flow at the Lake Erie terminal, however, is computed on the basis of flow continutiy and is not explicitly specified. Once flow boundary conditions are specified, equations developed in this report may be used to compute flow magnitudes and uncertainties at major branches of the rivers. Branches are identifed in the schematic by the rectangular boxes that include cross sections identifiers where flow is measured by the U.S. Army Corps of Engineers.

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By DAVID J. HOLTSCHLAG, and JOHN A. KOSCHIK, U.S. Army Corps of Engineers

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Lansing, Michigan
2001

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CONVERSION FACTORS

Multiply	By	To obtain
cubic foot (ft^3)	0.02832	cubic meter
feet (ft)	0.3048	meters
inches (in.)	25.4	millimeters
gallon (gal.)	3.785	liter
miles	1.609	kilometers

Temperature in degrees Celsius (°C) can be converted to degrees Fahrenheit (°F) as follows:

°F = 32 + (°C*1.8)

VERTICAL DATUM

The vertical datum currently used throughout the Great Lakes is the International Great Lakes Datum of 1985 (IGLD 1985), although references to the earlier datum of 1955 are still common. This datum is a dynamic height system for measuring elevation, which varies with the local gravitational force, rather than an orthometric system, which provides an absolute distance above a fixed point. The primary reason for adopting a dynamic height system within the Great Lakes is to provide an accurate measurement of potential hydraulic head. The reference zero for IGLD (1985) is a tide gage at Rimouski, Quebec, which is located near the outlet of the Great Lakes–St. Lawrence River system. The mean water-surface elevation at the Rimouski, Quebec, gage approximates mean sea level.

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Abstract

St. Clair and Detroit Rivers are connecting channels between Lake Huron and Lake Erie in the Great Lakes waterway, and form part of the boundary between the United States and Canada. St. Clair River, the upper connecting channel, drains 222,400 square miles and has an average flow of about 182,000 cubic feet per second. Water from St. Clair River combines with local inflows and discharges into Lake St. Clair before flowing into Detroit River. In some reaches of St. Clair and Detroit Rivers, islands and dikes split the flow into two to four branches. Even when the flow in a reach is known, proportions of flows within individual branches of a reach are uncertain. Simple linear regression equations, subject to a flow continuity constraint, are developed to provide estimators of these proportions and flows. The equations are based on 533 paired measurements of flow in 13 reaches forming 31 branches. The equations provide a means for computing the expected values and uncertainties of steady-state flows on the basis of flow conditions specified at the upstream boundaries of the waterway. In 7 upstream reaches, flow is considered fixed because it can be determined on the basis of flows specified at waterway boundaries and flow continuity. In these reaches, the uncertainties of flow proportions indicated by the regression equations can be used directly to determine the uncertainties of the corresponding flows. In the remaining 6 downstream reaches, flow is considered uncertain because these reaches do not receive flow from all the branches of an upstream reach, or they receive flow from some branches of more than one upstream reach. Monte Carlo simulation analysis is used to quantify this increase in uncertainty associated with the propagation of uncertainties from upstream reaches to downstream reaches. To eliminate the need for Monte Carlo simulations for routine calculations, polynomial regression equations are developed to approximate the variation in uncertainties as a function of flow at the headwaters of St. Clair River. Finally, monthly flow-duration data on the main channels of St. Clair and Detroit Rivers are used with the equations developed in this report to estimate the steady-state flow-duration characteristics of selected branches.

INTRODUCTION

The Michigan Department of Environmental Quality (MDEQ) Source Water Assessment Program (SWAP) together with the Detroit Water and Sewerage Department (DWSD) are assessing the vulnerability of public-water intakes to contamination on the St. Clair–Detroit River waterway. These intakes provide a water supply to about 4.2 million people in the Detroit, Michigan, area, as well as about 2 million others in Michigan and Canada. As part of this assessment, the U.S. Geological Survey (USGS) and the Detroit District of the U.S. Army Corps of Engineers (USACE) are developing a two-dimensional hydrodynamic model of the waterway. Results of this study facilitate the implementation of the SWAP and the development of the hydrodynamic model by providing a system of equations for computing the expected steady-state flow distribution within major branches of the waterway and describing the corresponding flow-duration characteristics.

Purpose and Scope

The purpose of this report is to facilitate the source water assessment of public-water intakes on St. Clair and Detroit Rivers by providing information on the steady-state flow distribution and flow duration of selected branches in the waterway. This report describes an analysis of 533 ADCP (acoustic Doppler current profiler) flow (discharge) measurements on 31 branches of St. Clair and Detroit Rivers obtained from May 1996 to June 2000 by the Detroit District of the U.S. Army Corps of Engineers (USACE). A system of equations is developed for computing the magnitude and uncertainty of flow proportions through selected branches. The information on flow proportions together with flows specified at the waterway boundaries provide a basis for estimating the magnitude and uncertainty of steady-state flows through the corresponding branches. Additional uncertainty associated with the propagation of variance from upstream to downstream reaches is estimated by use of Monte Carlo analysis. Regression equations are developed to estimate this increase in variance as a function of specified flow, and provide a means of estimating the variance of the flows within branches. Together with monthly flow-duration data on the main channels of St. Clair and Detroit Rivers, the equations are used to compute monthly flow-duration characteristics of selected branches in the St. Clair and Detroit Rivers.

Study Area

St. Clair River, Lake St. Clair, and Detroit River form a waterway that is part of the boundary between the United States and Canada (fig. 1). The waterway is a major navigational and recreational resource of the Great Lakes region that connects Lake Huron with Lake Erie. St. Clair River (the upper connecting channel) extends about 39 mi from its head at the outlet of Lake Huron near Port Huron, Michigan, to an extensive delta area. Throughout its length, water-surface elevations in St. Clair River decrease about 5 ft as it discharges an average of 182,000 ft^3/s from a drainage area of about 222,400 mi^2. Lake St. Clair receives water from St. Clair River, and lesser amounts from Clinton River in Michigan and Thames and Sydenham Rivers in Ontario, Canada. Along the 25-foot deep navigational channel, the lake is 35 miles long. The lake's round shape and shallow depths that average about 11 ft, make it highly susceptible to winds from all directions. Detroit River (the lower connecting channel) receives water from Lake St. Clair, and then flows 32 mi to Lake Erie. Water-surface elevations fall about 3 ft within Detroit River, which has an average discharge of about 186,000 ft^3/s.

Water level, wind, and flow conditions along the boundaries of the St. Clair–Detroit River control flow throughout the waterway. Ordinarily, these boundary conditions change so rapidly that, because of the momentum characteristics of the flow, conditions in the waterway do not reach steady state before additional changes occur at the boundaries. Thus, flow in the St. Clair–Detroit River technically is considered unsteady or transient. Although true steady-state flow conditions may be rare, this idealization is a useful approximation over relatively short periods of time and will be used in this report to simplify the description of flow characteristics within major branches of the waterway.

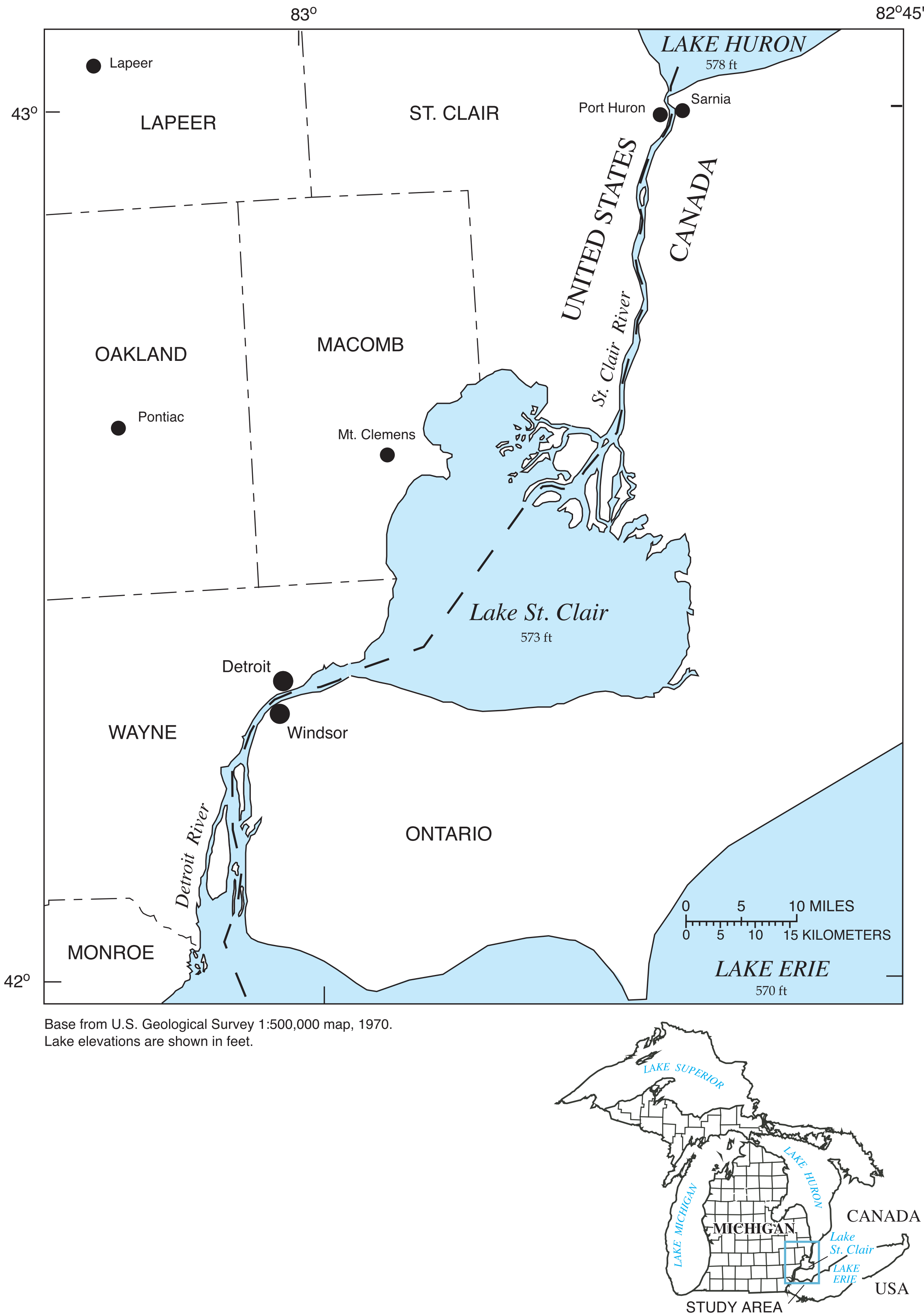

Figure 1. Study area showing the St. Clair and Detroit Rivers within the Great Lakes Waterway.

Flow-Measurement Data

The St. Clair–Detroit River waterway is a major water resource shared by the United States and Canada. To understand the hydraulic and hydrologic characteristics of the waterway, the United States and Canada established cross sections and began measuring flow in 1841 (Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data, 1994). In the United States, the Detroit District of the USACE is the lead agency for this data-collection effort. In 1996, the USACE upgraded it flow-measurement equipment to include acoustic Doppler current profilers (ADCP), which can be used to obtain flow measurements more quickly and accurately than conventional current-meter measurements (Morlock, 1995). Use of ADCP equipment facilitates the comparison of measurements by providing data at different location under similar hydraulic conditions.

USACE deploys ADCP equipment from a moving boat that generally can transect the channel in about 5 to 15 minutes. The reliability of these measurements commonly is verified by comparing flows measured during two or more transects, run in both directions across the channel. In this report, flow was computed as the average flow of all transects in which the accuracy of the measurement was subjectively rated as "fair," "good," or "excellent" based on the measurement conditions.

Measurement cross-sections are located along the main channels and at major branches associated with islands and dikes (figs. 2 and 3). On St. Clair River, ADCP measurements are routinely obtained at 22 cross sections including branches around Stag and Fawn Islands, and on the distributaries of St. Clair River in the St. Clair River Delta. On Detroit River, 24 cross sections are commonly measured on branches around Peche Island, Belle Isle, Fighting Island, Grosse Ile, and numerous smaller islands and dikes associated with navigational channels.

Measurement sets commonly are obtained at about 6-week intervals during the ice-free period of the year. In this report, 533 ADCP measurements obtained from May 1996 to June 2000 by USACE were selected for analysis (appendix A). This selection provided about 17 to 20 flow measurements for each branch. About 3 days are required to obtain a complete set of measurements at all cross-sections typically measured on St. Clair and Detroit Rivers; measurements sets on St. Clair and Detroit Rivers usually are obtained within 14 days of one another. In this report, 12 branches were selected on 5 reaches of St. Clair River (fig. 2) and 19 branches were selected on 8 reaches in Detroit River (fig. 3) to quantify flow-distribution and flow-duration characteristics.

FLOW-DISTRIBUTION ANALYSIS

This report defines 13 reaches in the St. Clair–Detroit River waterway where flow is partitioned into two or more branches (channels) by one or more islands. Flow within branches is defined at cross sections regularly measured by the USACE. For simplicity of notation, the branch will be identified by the corresponding cross-section number.

In some reaches, flow is considered fixed because the total flow in the reach is known on the basis of flows specified at the boundaries and the continuity constraint. In these reaches, the uncertainty of the flow estimate within each branch is determined strictly by the local uncertainty of the corresponding flow proportions. The magnitude and uncertainty of these proportions are described by 13 sets of regression equations developed in this report. In other reaches, however, flow is considered uncertain (fig. 4) because the inflow to the reach does not include all upstream branches or includes flow from some branches of more than one upstream reach. Here, the continuity constraint does not apply, so the uncertainty of flow proportions in upstream branches propagates downstream. This uncertainty is combined with the local uncertainty of the flow proportions in the downstream branches. The propagation of this uncertainty and its effect on the uncertainty of flow estimates was investigated by use of Monte Carlo analysis. Corresponding regression equations were developed to estimate the increase in variance as a function of specified flow to facilitate estimation of the uncertainty of the flow estimates.

Figure 2. Locations of selected flow-measurement cross sections on St. Clair River within the Great Lakes Waterway.

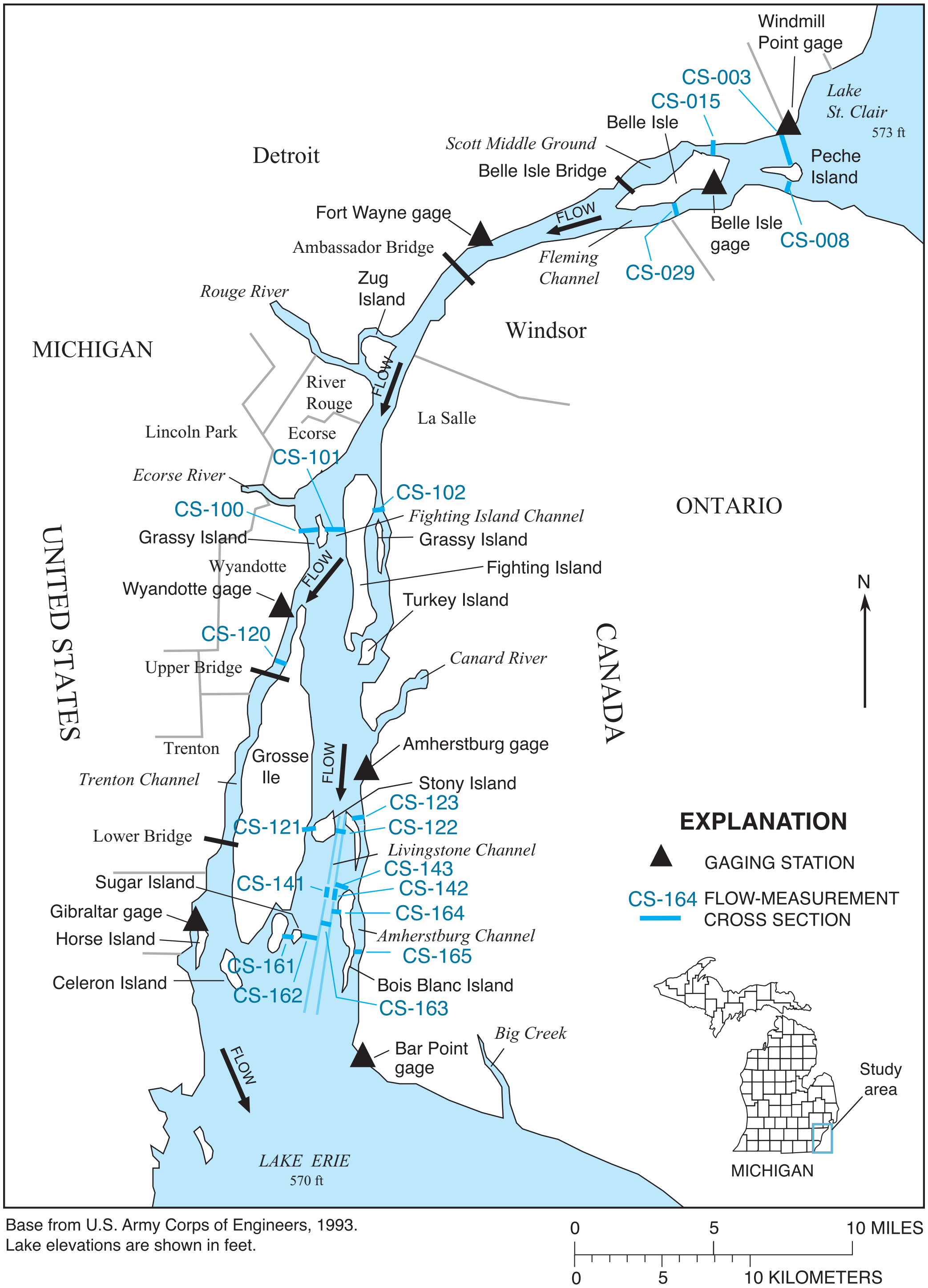

Figure 3. Locations of selected flow-measurement cross sections on Detroit River within the Great Lakes Waterway.

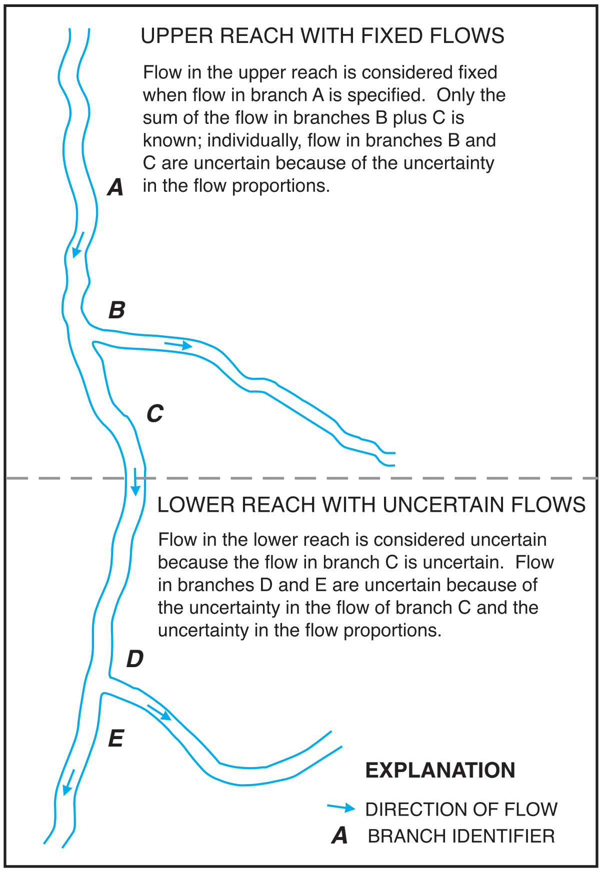

Figure 4. Schematic describing terminology for reaches with fixed or uncertain flows.

Estimation of Flow Proportions in Branches

Simple linear regression equations were developed to estimate the proportion of flow in individual branches around islands or dikes in the St. Clair–Detroit River waterway, either as a constant or as a function of the total flow in the reach. The difficulty with this approach is that the estimates of proportions can theoretically fall outside the interval from zero to one. Despite this potential problem, the regression estimator can provide useful estimates if the linear approximation occurs over a fairly narrow range of proportions and these proportions are not too close to zero or one, given the magnitude and the uncertainty of the estimate.

Most reaches contain a single island; thus, only two branches are formed, one on either side of the island. Other reaches contained multiple islands that create three or four branches. Most reaches received inflow from only one upstream reach; other reaches received inflow from branches of two upstream reaches. Estimation of flow proportions in simple, two-branch reaches will be discussed first, followed by its extension to three and four-branch reaches.

Two-Branch Reaches

In a two-branch reach, water flows around both sides of an island and converges immediately downstream of the island. To ensure continuity of flow, the total flow q_T above and below the island is computed as the sum of measured flow in the two branches, q_1 and q_2, even in areas where nearby measurements of the total flow were available on the main channel. The corresponding proportions of flow through each branch were computed as $p_1 = q_1/q_T$ and $p_2 = q_2/q_T = 1 - p$. In this report, an over arrow is used to denote a column vector such as the set of flow measurements, $\vec{q}$, or corresponding measured proportions, $\vec{p}$. The variable n_o is used to indicate the number of measurements.

The form of a simple linear regression equation is

$$p_1 = \beta_0 + \beta_1 q_T + \varepsilon_1 . \qquad (1)$$

The parameter vector $\vec{\beta}$, generally containing β_0 and β_1 was estimated by least-squares as

$$\hat{\vec{\beta}} = (X'X)^{-1} X' \vec{p}_1 \quad , \qquad (2)$$

where the nominal design matrix was $X = [\vec{1}_{n_o} \vec{q}_T]$, where the prime symbol indicates a matrix transpose operation, and the -1 superscript indicates matrix inversion. The regression residuals, $\hat{\vec{\varepsilon}} = \vec{p} - \hat{\vec{p}}$, were used to compute the sample error variance as $\hat{\sigma}_\varepsilon^2 = \hat{\sigma}_{CS}^2 = \hat{\vec{\varepsilon}}'\hat{\vec{\varepsilon}}/(n_o - n_p)$, where n_p is the number of parameters (length of $\hat{\vec{\beta}}$). The corresponding covariance of $\hat{\vec{\beta}}$ was computed as $\hat{C}_{\hat{\vec{\beta}}} = (X'X)^{-1} \hat{\sigma}_{CS}^2$.

The square roots of the main diagonal components of $\hat{C}_{\vec{\beta}}$ are the standard errors of the parameters. These standard errors were used to test the statistical significance of the model parameters at the 95-percent confidence level. When a parameter estimate was not significantly different than zero, the corresponding explanatory variable was dropped, reducing the length of $\hat{\vec{\beta}}$ and then parameter estimation was repeated with a simpler design matrix. When the flow proportion was estimated as a function of total flow, the uncertainty of the estimated proportion corresponding to a new flow indexed by i was computed as

$$\hat{\sigma}^2_{CS|Q_T = q_{T_i}} = \hat{\sigma}^2_{CS}\left(1 + \frac{1}{n_o} + \frac{(q_{T_i} - \bar{q}_T)^2}{(n_o - n_p)\sigma^2_{q_T}}\right). \quad (3)$$

Otherwise, the uncertainty of the estimated proportion was the sample error variance.

Three- and Four-Branch Reaches

Some reaches in the St. Clair–Detroit River waterway contain three or four branches. The method applied to two-branch reaches was extended to the number of branches represented by n_b. Because of the flow-continuity constraint, proportions in only $n_b - 1$ of the branches within a reach are linearly independent, and, therefore, capable of being uniquely estimated. Therefore, $n_b - 1$ of the branches were chosen (arbitrarily) for development of regression equations.

The general form of the extended regression equation is

$$\begin{bmatrix} p_{1,1} \\ p_{1,2} \\ \cdots \\ p_{1,n_o} \\ p_{2,1} \\ p_{2,2} \\ \cdots \\ p_{2,n_o} \\ \cdots \\ p_{n_b-1,1} \\ p_{n_b-1,2} \\ \cdots \\ p_{n_b-1,n_o} \end{bmatrix} = \begin{bmatrix} \beta_{0,1} + \beta_{1,1} \cdot q_{T,1} \\ \beta_{0,1} + \beta_{1,1} \cdot q_{T,2} \\ \cdots \\ \beta_{0,1} + \beta_{1,1} \cdot q_{T,n_o} \\ \beta_{0,2} + \beta_{1,2} \cdot q_{T,1} \\ \beta_{0,2} + \beta_{1,2} \cdot q_{T,2} \\ \cdots \\ \beta_{0,2} + \beta_{1,2} \cdot q_{T,n_o} \\ \cdots \\ \beta_{0,n_b-1} + \beta_{1,n_b-1} \cdot q_{T,1} \\ \beta_{0,n_b-1} + \beta_{1,n_b-1} \cdot q_{T,2} \\ \cdots \\ \beta_{0,n_b-1} + \beta_{1,n_b-1} \cdot q_{T,n_o} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,1} \\ \varepsilon_{1,2} \\ \cdots \\ \varepsilon_{1,n_o} \\ \varepsilon_{2,1} \\ \varepsilon_{2,2} \\ \cdots \\ \varepsilon_{2,n_o} \\ \cdots \\ \varepsilon_{n_b-1,1} \\ \varepsilon_{n_b-1,2} \\ \cdots \\ \varepsilon_{n_b-1,n_o} \end{bmatrix} \quad (4)$$

The $n_o \times (n_b - 1)$ column vector $\vec{\varepsilon}$ was manipulated to create individual residual vectors of length n_o that represented individual branches, and concatenating the vectors to form the matrix $[\vec{\varepsilon}_1' \ \vec{\varepsilon}_2' \cdots \vec{\varepsilon}_{n_b-1}']'$. Parameters for equation 4 were estimated simultaneously by minimizing the sum of squared residuals, $\vec{\varepsilon}'\vec{\varepsilon}$. The estimation was carried out by use of a function in MATLAB (The Mathworks, Inc., 1999a) that was written for each reach (appendix B) and using the MATLAB function *nlinfit* (The Mathworks, Inc., 1999b, page 2-162) to minimize the sum of squared residuals. Model parameters were tested to determine whether they were significantly different than zero at the 95-percent confidence level. If necessary, the design matrix was simplified to eliminate statistically insignificant components in the design matrix. The regression residuals were used to estimate the covariance structure of the model errors as

$$\hat{C}_{\varepsilon_1, \varepsilon_2, \ldots \varepsilon_{n_b-1}} = [\hat{\varepsilon}_1 \hat{\varepsilon}_2 \ldots \hat{\varepsilon}_{n_b-1}]' \begin{bmatrix} \hat{\varepsilon}'_1 \\ \hat{\varepsilon}'_2 \\ \ldots \\ \hat{\varepsilon}'_{n_b-1} \end{bmatrix} / (n_o - 1) \quad (5)$$

The estimated variance $\hat{\sigma}^2_{n_b}$ of the proportion of flow in the excluded branch $\hat{p}_{n_b}$ was computed as the sum of elements in the estimated covariance matrix $\hat{C}_{\varepsilon_1, \varepsilon_2, \ldots \varepsilon_{n_b-1}}$. Individual (nonsimultaneous) confidence intervals previously described were used to assess the statistical significance of all model parameters, and to estimate the uncertainty of flow proportions and corresponding flows. Equations for computing flow proportions are provided in table 1. The relation between computed and estimated flow proportions is shown in figs. 5–7 for selected reaches. No bias or heteroscedasticity was detected in the residuals of the regression equations (figs. 8–9).

Branch Flow in Reaches with Fixed Total Flows

Flows in a reach are considered fixed if flow continuity restricts the total flow in all the branches within a reach to a value specified at the boundaries. In reaches with fixed flow, the uncertainty of the estimated proportions can be used to compute the uncertainties of the flows in individual branches. Alternatively, if the flow in all branches of a reach are measured, the total measured flow may be considered fixed and used to assess whether the (proportion of) flows measured in individual branches is consistent with the historical data used to develop the flow equations. In this way, the equations for computing flow proportions can be used to help quality assure flow measurements.

Flow-boundary conditions may be specified at the headwaters of St. Clair River, Q_{SCR}, and local inflows at the mouths of major tributaries including Black, Pine, and Belle Rivers on St. Clair River, Clinton, Thames, and Sydenham Rivers on Lake St. Clair, net inflow over Lake St. Clair, and River Rouge on Detroit River. Because flow at the headwaters of St. Clair River is much larger than the local inflows, local inflows are assigned constant average values (table 2).

Flow at the headwaters of St. Clair River can be specified or computed on the basis of a stage-fall-flow equation developed by USACE (J.A. Koschik, USACE, written commun., 1999) of the form:

$$Q_{SCR} = 81,359(Stage_{MBR} - 524.93)^2 \cdot (Stage_{FG} - Stage_{MBR})^{0.5}, \quad (6)$$

where Q_{SCR} is the computed flow at the headwaters of St. Clair River in cubic feet per second, $Stage_{FG}$ is the stage (water level) at Fort Gratiot, which is recorded by the NOS (National Ocean Service, a branch of the U.S. Department of Commerce, National Oceanic and Atmospheric Administration) gage 9014090, and $Stage_{MBR}$ is the stage at the mouth of Black River, recorded at gage 9014096, in international feet above the IGLD (International Great Lakes Datum) of 1955. Once flow is specified at the boundaries, expected flows and uncertainties for reaches with fixed flow can be computed by use of the equations in table 1.

Table 1. Equations for computing flow proportions in St. Clair and Detroit Rivers within the Great Lakes Waterway

[--, no data]

Reach name	Branch name	Cross-section number	Form of flow proportion equation (q_T is the total flow in the corresponding reach)	Intercept, $\hat{\beta}_0$ (Half width of 95-percent confidence interval)	Flow coefficient $\hat{\beta}_1$ (Half width of 95-percent confidence interval)	N	Standard error of the estimate	Mean of measured flow, in cubic feet per second (standard deviation)	Estimated correlation between model errors
Stag Island	Stag Island East	CS-208	$\hat{p}_{CS-208} = \hat{\beta}_0 + \hat{\beta}_1 q_T$	0.25185 (0.040645)	3.9158×10^{-7} (1.9860×10^{-7})	17	$6.3394 \cdot 10^{-3}$	67,770 (7,032)	--
	Stag Island West	CS-210	$\hat{p}_{CS-210} = 1 - \hat{p}_{CS-208}$	--	--	--	$6.3394 \cdot 10^{-3}$	136,210 (10,167)	--
	Combined	--	--	--	--	18	--	203,980 (17,009)	--
Fawn Island	Fawn Island West	CS-216	$\hat{p}_{CS-216} = \hat{\beta}_0 + \hat{\beta}_1 q_T$	0.87708 (0.02558)	-2.8634×10^{-7} (1.2708×10^{-7})	19	$4.7136 \cdot 10^{-3}$	164,230 (14,106)	--
	Fawn Island East	CS-218	$\hat{p}_{CS-218} = 1 - \hat{p}_{CS-216}$	--	--	--	$4.7136 \cdot 10^{-3}$	36,244 (4,463)	--
	Combined	--	--	--	--	--	--	200,475 (18,444)	--
Walpole Island	Chenal Ecarte	CS-222	$\hat{p}_{CS-222} = \hat{\beta}_1 q_T$	--	2.1066×10^{-7} (8.110×10^{-9})	17	$3.2261 \cdot 10^{-3}$	8,809 (1,431)	--
	Lower St. Clair River	CS-230	$\hat{p}_{CS-230} = 1 - \hat{p}_{CS-222}$	--	--	17	$3.2261 \cdot 10^{-3}$	195,170 (13,571)	--
	Combined	--	--	--	--	--	--	203,980 (14,829)	--
Dickinson-Harsens Island	South Channel	CS-232	$\hat{p}_{CS-232} = \hat{\beta}_0 + \hat{\beta}_1 q_T$	0.54859 (0.07260)	-4.6909×10^{-7} (3.7823×10^{-7})	16	0.011858	87,727 (5,775)	$\hat{\rho}_{CS-232, CS-240} = -0.529978$
	North Channel	CS-240	$\hat{p}_{CS-240} = \hat{\beta}_0 + \hat{\beta}_1 q_T$	0.24258 (0.07260)	$5.9493 \cdot \times 10^{-7}$ ($3.7823 \cdot \times 10^{-7}$)	16	0.008936	68,353 (7,174)	--
	Middle Channel	CS-242	$\hat{p}_{CS-242} = 1 - \hat{p}_{CS-232} - \hat{p}_{CS-240}$	--	--	--	0.010047	35,339 (3,017)	--
	Combined	--	--	--	--	--	--	191,419 (14,682)	--
Bassett-Seaway Island	Bassett Channel	CS-234	$\hat{p}_{CS-234} = 1 - \hat{p}_{CS-236} - \hat{p}_{CS-238}$	--	--	--	0.010000	6,991 (1,034)	--
	St. Clair Cutoff	CS-236	$\hat{p}_{cs-236} = \hat{\beta}_0$	0.54025 (0.1257)	--	17	0.022840	46,842 (4,026)	$\hat{\rho}_{CS-236, CS-238} = -0.94082$
	St. Clair Flats	CS-238	$\hat{p}_{cs-238} = \hat{\beta}_0$	0.37917 (0.01258)	--	17	0.027820	32,834 (2,925)	--
	Combined	--	--	--	--	--	--	86,667 (5,907)	--

Table 1. Equations for computing flow proportions in St. Clair and Detroit Rivers within the Great Lakes Waterway—*Continued*

Reach name	Branch name	Cross-section number	Form of flow proportion equation (q_T is the total flow in the corresponding reach)	Intercept, $\hat{\beta}_0$ (Half width of 95-percent confidence interval)	Flow coefficient $\hat{\beta}_1$ (Half width of 95-percent confidence interval)	N	Standard error of the estimate	Mean of measured flow, in cubic feet per second (standard deviation)	Estimated correlation between model errors
Peche Island	Peche Island North	CS-003	$\hat{p}_{cs-003} = \hat{\beta}_0$	0.73503 (0.00455)	--	19	0.009451	158,070 (15,037)	--
	Peche Island South	CS-008	$\hat{p}_{CS-008} = 1 - \hat{p}_{CS-003}$	--	--	--	0.009451	56,972 (5,680)	--
	Combined	--	--	--	--	--	--	215,045 (20,193)	--
Belle Isle	Scott Middle Ground	CS-015	$\hat{p}_{CS-015} = \hat{\beta}_0 + \hat{\beta}_1 q_T$	0.21262 (0.04736)	5.0612×10^{-7} (2.1984×10^{-7})	19	0.007104	69,189 (7,062)	--
	Fleming Channel	CS-029	$\hat{p}_{CS-029} = 1 - \hat{p}_{CS-015}$	--	--	--	0.007104	145,710 (9,287)	--
	Combined	--	--	--	--	--	--	214,901 (16,069)	--
Fighting- Grassy Island	American Grassy Island	CS-100	$\hat{p}_{CS-100} = \hat{\beta}_0$	0.26130 (0.00312)	--	18	0.007224	55,527 (6,732)	$\hat{\rho}_{CS-100, CS-101} = -0.76025$
	Fighting Island Channel	CS-101	$\hat{p}_{CS-101} = \hat{\beta}_0 + \hat{\beta}_1 q_T$	0.55823 (0.02979)	-1.8722×10^{-7} (1.3954×10^{-7})	18	0.005641	109,990 (11,043)	--
	Canadian Grassy Island	CS-102	$\hat{p}_{CS-102} = 1 - \hat{p}_{CS-100} - \hat{p}_{CS-101}$	--	--	--	0.004693	46,804 (5,534)	--
	Combined	--	--	--	--	--	--	212,320 (23,002)	--
Grosse Ile- Stony	Trenton Channel	CS-120	$\hat{p}_{CS-120} = \hat{\beta}_0$	0.22156 (0.00485)	--	17	0.0099257	46,787 (5,472)	$\hat{\rho}_{CS-120, CS-121} = 0.39854$
Stony Island-Dike	Grosse Ile-Stony Island	CS-121	$\hat{p}_{CS-121} = \hat{\beta}_0 + \hat{\beta}_1 q_T$	-0.11248 (0.05618)	9.0125×10^{-7} (2.6550×10^{-7})	17	0.010892	16,649 (5,412)	$\hat{\rho}_{CS-120, CS-122} = -0.75693$
	Upper Livingstone	CS-122	$\hat{p}_{CS-122} = \hat{\beta}_0 + \hat{\beta}_1 q_T$	0.34521 (0.05618)	-4.4833×10^{-7} 2.6548×10^{-7}	17	0.0088991	52,703 (3,471)	$\hat{\rho}_{CS-121, CS-122} = -0.66573$
	Upper Amherst-burg	CS-123	$\hat{p}_{CS-123} = \begin{cases} 1 - p_{CS-120} \\ -\hat{p}_{CS-121} - \hat{p}_{CS-122} \end{cases}$	--	--	--	0.010815	94,692 (6,206)	--
	Combined	--	--	--	--	--	--	210,831 (18,839)	--

Table 1. Equations for computing flow proportions in St. Clair and Detroit Rivers within the Great Lakes Waterway—*Continued*

Reach name	Branch name	Cross-section number	Form of flow proportion equation (q_T is the total flow in the corresponding reach)	Intercept, $\hat{\beta}_0$ (Half width of 95-percent confidence interval)	Flow coefficient $\hat{\beta}_1$ (Half width of 95-percent confidence interval)	N	Standard error of the estimate	Mean of measured flow, in cubic feet per second (standard deviation)	Estimated correlation between model errors
Bois Blanc Island	Amherstburg Gap	CS-143	$\hat{p}_{CS-143} = \hat{\beta}_0$	0.24095 (0.04091)	--	16	0.018619	22,365 (2,567)	--
	Lower Amherst-burg	CS-165	$\hat{p}_{CS-165} = 1 - \hat{p}_{CS-143}$	--	--	--	0.018619	70,571 (7,111)	--
	Combined	--	--	--	--	--	--	92,935 (8,726)	--
Bois Blanc Livingstone	Livingstone Gap East	CS-142	$\hat{p}_{CS-142} = \hat{\beta}_0$	0.59822 (0.04603)	--	17	0.089521	13,935 (2,629)	--
	Bois Blanc-Dike	CS-164	$\hat{p}_{CS-164} = 1 - \hat{p}_{CS-142}$	--	--	--	0.089521	8,650 (2,170)	--
	Combined	--	--	--	--	--	--	21,585 (2,762)	--
Livingstone Gap	Livingstone Gap West	CS-141	$\hat{p}_{CS-141} = \hat{\beta}_0$	0.35606 (0.01766)	--	18	0.035504	22,888 (2,329)	--
	Lower Livingstone	CS-163	$\hat{p}_{CS-163} = 1 - \hat{p}_{CS-141}$	--	--	--	0.035504	41,555 (4,355)	--
Sugar Island	Sugar Island West	CS-161	$\hat{p}_{CS-161} = \hat{\beta}_0 + \hat{\beta}_1 q_T$	0.63465 (0.05133)	1.8888×10^{-6} (1.2478×10^{-6})	17	0.017062	28,890 (5,943)	--
	Sugar Island East	CS-162	$\hat{p}_{CS-162} = 1 - \hat{p}_{CS-161}$	--	--	--	0.017062	11,595 (1,740)	--
	Combined	--	--	--	--	--	--	40,485 (7,526)	--

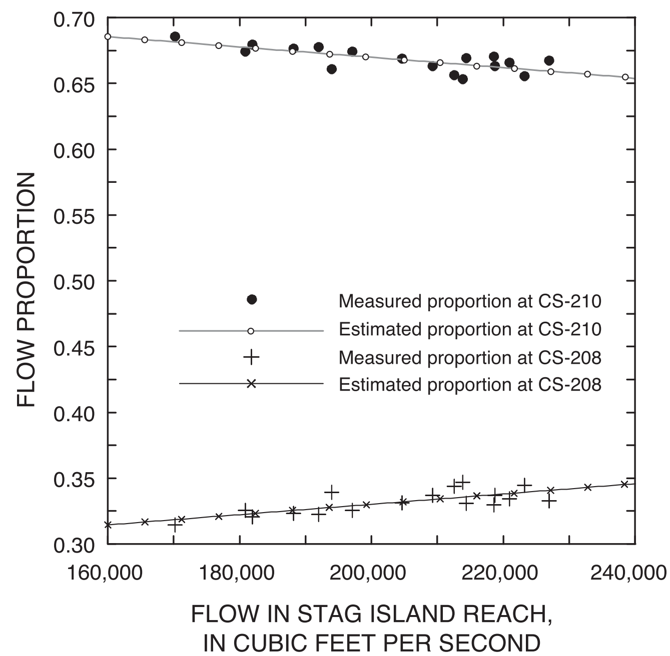

Figure 5. Relation between flow in the Stag Island reach and flow proportions in branches near cross-sections CS-208 and CS-210 on St. Clair River within the Great Lakes Waterway.

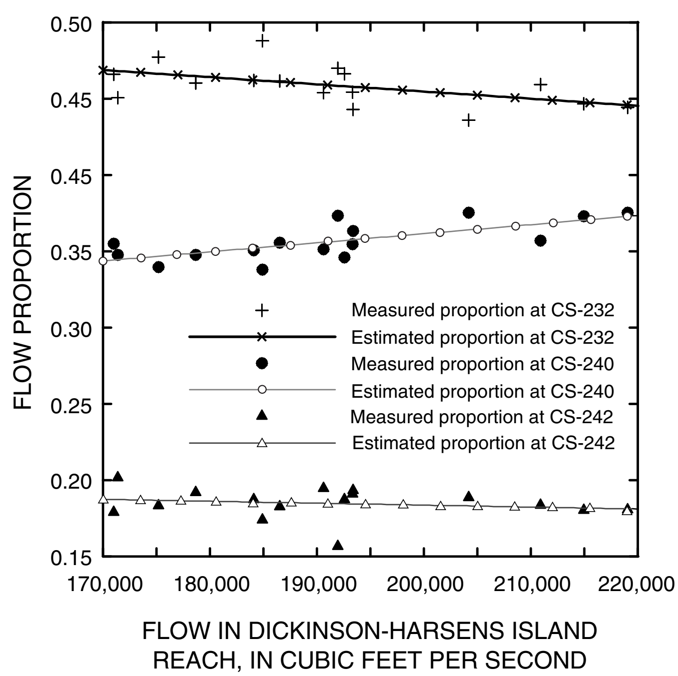

Figure 6. Relation between flow in the Dickinson-Harsens Island reach and estimated and measured flow proportions in branches near cross-sections CS-232, CS-240, and CS-242 on St. Clair River within the Great Lakes Waterway.

The following example for flow at CS-100, CS-101, and CS-102 illustrates the application of the equations in table 1. In this example, it will be assumed that the total inflow at the headwaters of St. Clair River and all minor tributaries to the reach, q_T, is 180,000 ft^3/s.

The equation for estimating the proportion of flow at CS-100 indicates that the proportion is a constant with parameter $\beta_0 = 0.2613$, thus, $\hat{p}_{CS-100} = 0.2613$ for all flows. Here, the total flow, $q_T = q_{CS-100} + q_{CS-101} + q_{CS-100}$, in all branches within the reach is fixed. Because there is no dependence on flow, the standard error of the estimated flow proportion at CS-100, $\hat{\sigma}_{CS-100}$, is a constant equal to 0.007224. This implies that the probability (Pr) that the true proportion $P_{\text{CS-100}}$ of flow in CS-100 is contained within the random interval shown below is 95 percent:

$$Pr[(\hat{p}_{CS-100} - t_{\alpha,df}\hat{\sigma}_{CS-100}) \le P_{CS-100} \le (\hat{p}_{CS-100} - t_{\alpha,df}\hat{\sigma}_{CS-100})] = 0.95,$$

where $t_{\alpha,df}$ is the value from the Student's *t*-distribution associated with probability α and *df* degrees of freedom. In this application, $t_{0.05,8-1} = 2.11$, and the corresponding interval is

$$Pr[0.2456 \le P_{CS-100} \le 0.2770] = 0.95.$$

For a fixed total flow Q_T equal to q_T, the expected flow in the corresponding branch can be computed as $\hat{q}_{CS-100} = q_T \cdot \hat{p}_{CS-100}$, with a corresponding standard error of

$$\hat{\sigma}_{[\hat{q}_{CS-100} | Q_T = q_T]} = q_T \cdot \hat{\sigma}_{\hat{p}_{CS-100}}.$$

Using this estimated standard error and the specified q_T value of 180,000 ft^3/s, the random interval likely to contain the true flow in the branch associated with cross section CS-100 is

$$Pr[44,300\ \text{ft}^3/\text{s} \le Q_{CS-100} \le 49,800\ \text{ft}^3/\text{s}] = 0.95.$$

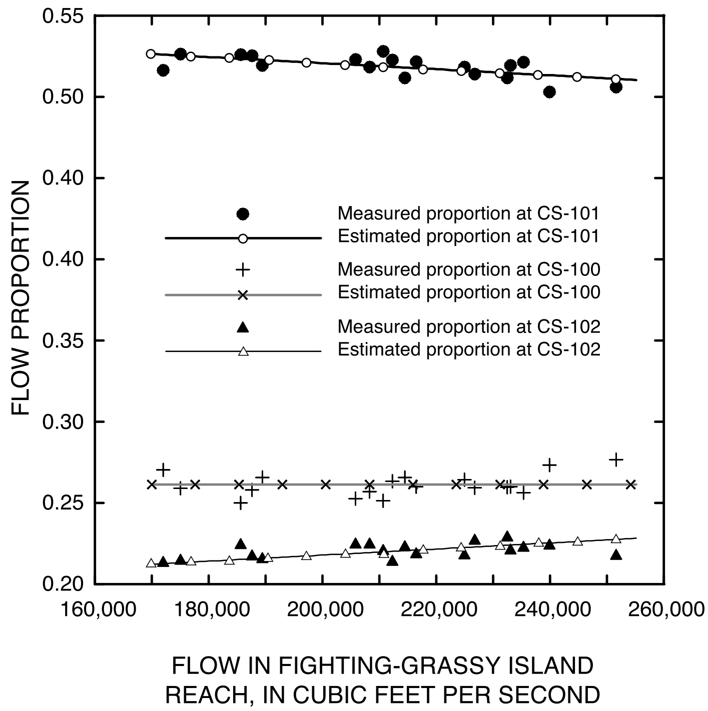

Figure 7. Relation between flow in the Fighting-Grassy Island reach and flow proportions in branches near cross sections CS-100, CS-101, and CS-102 on Detroit River within the Great Lakes Waterway.

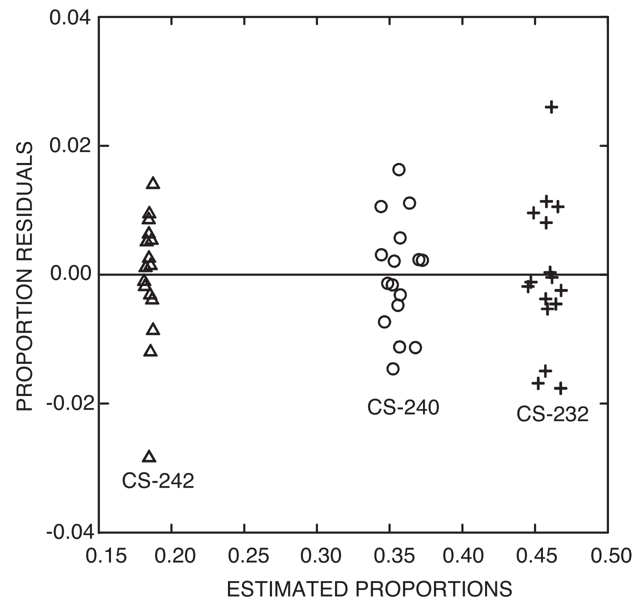

Figure 8. Relation between estimated proportions and residuals at cross-sections CS-232, CS-240, and CS-242 near Dickinson-Harsens Island reach on St. Clair River within the Great Lakes Waterway.

In contrast to CS-100, the proportion of flow in CS-101 is a linear function of the total flow as

$$\hat{p}_{CS-101}|q_T = 0.55823 - 1.8722 \cdot 10^{-7} \cdot q_T .$$

Again, for a hypothetical total flow of 180,000 ft^3/s, the expected proportion at CS-101 computed by use of the preceding equation is 0.5245. The estimated standard error of the proportion is computed by use of equation 3 and data in table 1 as

$$\hat{\sigma}[\hat{p}_{CS-100}] = \begin{cases} \hat{\sigma}_{CS-101}\sqrt{\left(1+\frac{1}{n}+\frac{(q_T-\bar{q}_T)^2}{(n-1)\cdot\sigma_{q_T}^{\ 2}}\right)} \\ 0.005641\sqrt{1+\frac{1}{18}+\frac{(180,000-212,32\ )^2}{(18-1)\cdot 23002^2}} \\ 0.006106 \end{cases} .$$

The 95-percent confidence interval on the flow proportion at CS-101 is $Pr[0.5116 \le P_{CS-101} \le 0.5374] = 0.95$.

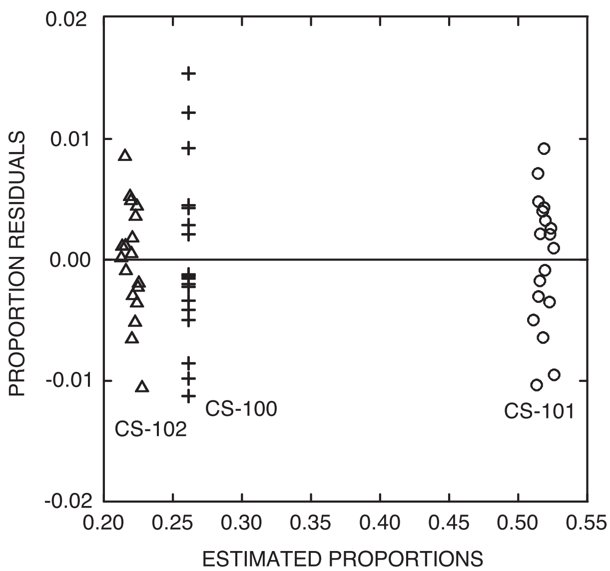

Figure 9. Relation between estimated proportions and residuals at cross-sections CS-100, CS-101, and CS-102 near Fighting-Grassy Island reach on Detroit River within the Great Lakes Waterway.

Table 2. Average local inflows at selected boundaries to the St. Clair-Detroit River Waterway

Description of local inflow source	**Waterway component receiving inflow**	**Approximate drainage area (square miles)**	**Approximate average flow (cubic feet per second)**
Black River..............	St. Clair River	746	489
Pine River................	St. Clair River	194	119
Belle River	St. Clair River	777	478
Sydenham River.......	Lake St. Clair	2,043	1,861
Clinton River...........	Lake St. Clair	1,206	928
Thames River	Lake St. Clair	4,330	4,857
Net lake inflow (Atmospheric, and surface and ground-water sources)................	Lake St. Clair	670	626
River Rouge	Detroit River	467	312

The corresponding estimate of flow through branch CS-101 for a total flow of 180,000 ft^3/s is 94,410 ft^3/s, with a 95-percent confidence interval that extends from 92,088 to 96,732 ft^3/s.

Finally, the uncertainty of the flow proportion at CS-102 is based on the variance of flows at CS-100 and CS-101 and their correlation. Specifically, the correlation matrix of errors in proportions at CS-100 and CS-101 is

$$\hat{R}_{p_{CS-100},\,p_{CS-101}} = \left\{ \begin{array}{l} \begin{bmatrix} 1 & \dfrac{\hat{C}_{p_{CS-100},\,p_{CS-101}}}{\hat{\sigma}_{p_{CS-100}}\hat{\sigma}_{p_{CS-101}}} \\ \dfrac{\hat{C}_{p_{CS-100},\,p_{CS-101}}}{\hat{\sigma}_{p_{CS-100}}\hat{\sigma}_{p_{CS-101}}} & 1 \end{bmatrix} \\ \begin{bmatrix} 1 & \hat{\rho}_{CS-100,\,CS-101} \\ \hat{\rho}_{CS-100,\,CS-101} & 1 \end{bmatrix} \end{array} \right. , \quad (7)$$

where $\hat{\rho}_{CS-100,\,CS-101} = -0.76025$. The covariance at q_T equal to 180,000 ft^3/s is estimated by use of the estimated correlation matrix and the variance estimates of the proportions at q_T as

$$\hat{C}_{p_{CS-100},\,p_{CS-101}}\Big|_{q_T = 180,000\,ft^3 \cdot s^{-1}} = \begin{bmatrix} \hat{\sigma}_{\hat{q}_{CS-100}} & 0 \\ 0 & \hat{\sigma}_{\hat{q}_{CS-101}} \end{bmatrix}\Bigg|_{Q_T} \cdot \hat{R}_{p_{CS-100},\,p_{CS-101}} \cdot \begin{bmatrix} \hat{\sigma}_{\hat{q}_{CS-100}} & 0 \\ 0 & \hat{\sigma}_{\hat{q}_{CS-101}} \end{bmatrix}\Bigg|_{Q_T}$$

$$= 10^{-4} \cdot \begin{bmatrix} 0.5509 & -0.3445 \\ -0.3445 & 0.3728 \end{bmatrix} .$$

The estimated standard error for the flow proportion at CS-102 is the square root of the sum of elements in the estimated covariance matrix at q_T=180,000, which equals 0.004844. The estimated proportion of flow at CS-102 equals

$$1 - \hat{p}_{CS-100} - \hat{p}_{CS-101} = 0.2142 .$$

The 95-percent confidence interval for this proportion extends from 0.2049 to 0.2244. The corresponding estimated flow is 38,566 ft^3/s, with a 95-percent confidence interval that extends from 36,726 to 40,406 ft^3/s.

Branch Flow in Reaches with Uncertain Total Flows

Flows in a reach may be uncertain because inflows do not include flow from all the branches in an upstream reach or because some branches from more than one upstream reach are included. In addition, the total flow may be uncertain because of the cascade of uncertainty from the inflows themselves. The uncertainty in total flow adds uncertainty to the computed flows within individual branches that is not included in the uncertainty of local flow proportions. This section describes the method used to assess and predict the increase in variance associated with uncertainty in reach flows.

Monte Carlo Analysis of Flow Uncertainties

Monte Carlo analysis is a statistical technique that uses random sampling and simulation techniques to quantify the probabilistic characteristics of complex or nonlinear random phenomena. In this report, Monte Carlo analysis is used to investigate the uncertainty of flows computed as the product of two or more flow proportions and a specified flow. Multiplication of estimated flow proportions, which contain a random component, is a nonlinear transformation of random variables that are difficult to approximate by use of linear statistical methods. In addition to the difficulty of estimating the variance of the resulting product, the distribution of this uncertainty is not necessarily normal (Gaussian), even if the individual proportions are normally distributed. The number of random components generated determines the precision of a Monte Carlo analysis. In this report, the number (length) of the random components n_{sim} was 1 million.

Monte Carlo analysis was used to estimate the variance associated with the uncertainty of upstream flows and compares it with the variance associated with the local flow proportions. In addition, nonparametric confidence intervals are compared with parametric confidence intervals to assess the adequacy of a normal distribution to describe the distribution of the uncertainty. The discussion begins with an outline of the Monte Carlo analysis for a simple hypothetical pair of upstream and downstream reaches and ends with implementation details for more complex situations. A listing of the code developed for the Monte Carlo analysis is in appendix C.

Consider a hypothetical pair of reaches (fig. 4) in which the upstream reach includes a specified flow at branch *A*, Q_A, that discharges into branches *B* and *C*. Only flow from branch *C*, however, discharges into the downstream reach that includes branches *D* and *E*. To describe the uncertainty of flows in the downstream reach, a random sample of flow proportions is generated for branch *C*. The distribution of the proportions is determined by the characteristics of the corresponding regression equation with parameters for intercept and flow dependence indicated by $\hat{\beta}_{0(C)}$ and $\hat{\beta}_{1(C)}$, respectively; the possibly flow-dependent uncertainty of the proportions, $\hat{\sigma}_{p(C)}|_{Q_A}$; and the particular random sample $\vec{\hat{\upsilon}}_1$ generated from a *t*-distribution (with degrees of freedom equal to those in the corresponding regression equation), as

$$\begin{aligned} \vec{\hat{p}}_C &= \beta_{0(C)} + \beta_{1(C)} \cdot Q_A + \vec{\upsilon}_1 \cdot \hat{\sigma}_{p_C|Q_A} \\ \vec{\hat{q}}_C &= \vec{\hat{p}}_C \cdot Q_A \end{aligned} \quad . \qquad (8)$$

The random sample of flows in branch *C*, $\vec{\hat{q}}_C$, then is used to generate corresponding samples of proportions and flows at branches *D* and *E*. One sample uses the mean of $\vec{\hat{q}}_C$ and a second random component

$\tilde{\vec{\upsilon}}_2$ to describe the uncertainties of proportions and flows in branches *D* and *E* as determined only by the uncertainties of the corresponding equations for estimating flow proportions. The second sample uses the individual values of $\bar{\vec{q}}_C$ and the random component $\tilde{\vec{\upsilon}}_2$ to describe the uncertainties of proportions and flows in branches *D* and *E* as determined by both the uncertainties of upstream flows and the uncertainties of the corresponding equations for estimating flow proportions. These equations written for flow in branch *E* are

$$\begin{aligned}
\bar{\bar{\vec{p}}}_E &= \beta_{0(E)} + \beta_{1(E)} \cdot \mathrm{mean}(\bar{\vec{q}}_C) + \vec{\upsilon}_2 \cdot \hat{\sigma}_{p_E|\mathrm{mean}(\bar{\vec{q}}_C)} \\
\tilde{\bar{\vec{p}}}_E &= \beta_{0(E)} + \beta_{1(E)} \cdot \bar{\vec{q}}_C + \vec{\upsilon}_2 \cdot\cdot \hat{\vec{\sigma}}_{p_E|\bar{q}_C} \\
\bar{\bar{\vec{q}}}_E &= \bar{\bar{\vec{p}}}_E \cdot \mathrm{mean}(\bar{\vec{q}}_C) \\
\tilde{\bar{\vec{q}}}_E &= \tilde{\bar{\vec{p}}}_E \cdot\cdot \bar{\vec{q}}_C
\end{aligned} \tag{9}$$

where "$\cdot\cdot$" denotes an element-by-element multiplication of vector components.

Reaches with more than two branches required random components that were consistent with both the sample correlation structure and the possibly flow-dependent uncertainty of the model errors. To develop this error structure, the standard error of prediction was computed for the estimated proportion and flow magnitude within each branch. Then, using the sample error correlation shown in table 1, the prediction error covariance was computed as

$$\hat{C}_{\hat{p}_1, \hat{p}_2 \cdots \hat{p}_{n_b-1}} = \begin{cases} diag[\hat{\sigma}_{1|q_T}, \hat{\sigma}_{2|q_T}, \ldots, \sigma_{n_{b-1|q_T}}] \\ \cdot \hat{R}_{\hat{p}_1, \hat{p}_2 \cdots \hat{p}_{n_b-1}} \\ \cdot\, diag[\hat{\sigma}_{1|q_T}, \hat{\sigma}_{2|q_T}, \ldots, \sigma_{n_{b-1|q_T}}] \end{cases} . \tag{10}$$

Then, an independent *t*-distributed random matrix $\underline{\upsilon}$ of size $n_{sim} \times (n_b - 1)$ was generated and multiplied by the square root of the sample covariance matrix to compute the correlated random components with appropriately scaled variances as

$$\underline{\xi} = \underline{\upsilon}\sqrt{\hat{C}_{\hat{p}_1, \hat{p}_2 \cdots \hat{p}_3}} . \tag{11}$$

Vectors from the correlated random matrix $\underline{\xi}$ were used in place of the uncorrelated random vectors $\tilde{\vec{\upsilon}}$ in equation 9 to generate proportions and corresponding flows.

Eight sets of Monte Carlo simulations were conducted to evaluate the effect of the uncertainty of flow in selected reaches on the uncertainty of flow in the affected branches. Specified flows ranged from 170,000 to 230,000 ft^3/s in increments of 10,000 ft^3/s at the headwaters of St. Clair River; flows at other boundaries were held constant at values shown in table 2. Each simulation was based on random vector lengths of 1 million. Average values and lower and upper nonparametric confidence intervals, computed by use of 2.5 and 97.5 percentiles of flow proportions and flows generated from the Monte Carlo analysis, were used to compute nonparametric 95-percent confidence intervals. In addition, ratios of the variance $\bar{\bar{\vec{q}}}_E$, which reflects the uncertainty of the reach flows and downstream (local) flow proportions, to the variance of $\tilde{\bar{\vec{q}}}_E$, which reflects only the uncertainty of downstream flow proportions, were computed. In reaches with only two branches, the flow continuity constraint was used to compute the remaining flow proportion estimates. Results for a Monte Carlo analysis in which flow at the headwaters of St. Clair River was specified to be 180,000 ft^3/s are provided in table 3.

Table 3. Summary of proportions and flows on selected branches of St. Clair and Detroit Rivers within the Great Lakes Waterway from Monte Carlo simulation

[%, percent; --, no data]

Branch	Cross section	Flow variability	Flow proportion			Flow (cubic feet per second)		
			Average	95% confidence limit		Average	95% confidence limit	
				Lower	Upper		Lower	Upper
ST. CLAIR RIVER								
Stag Island East	CS-208	Fixed	0.3225	0.3078	0.3372	58,212	55,554	60,868
Stag Island West	CS-210	Fixed	.6775	.6628	.6922	122,277	55,554	124,935
Fawn Island West	CS-216	Fixed	.8252	.8147	.8357	149,437	55,554	151,337
Fawn Island East	CS-218	Fixed	.1748	.1643	.1853	31,649	55,554	33,548
Chenal Ecarte	CS-222	Fixed	.0382	.0306	.0457	6,908	55,554	8,270
Lower St. Clair River	CS-230	Fixed	.9619	.9543	.9694	174,178	55,554	175,539
South Channel	CS-232	Fixed	.4669	.4396	.4943	81,327	55,554	86,097
		Uncertain	--	--	--	81,327	55,554	86,121
North Channel	CS-240	Fixed	.3462	.3256	.3668	60,299	55,554	63,892
		Uncertain	--	--	--	60,300	55,554	63,941
Middle Channel	CS-242	Fixed	.1869	.1629	.2108	32,552	55,554	36,713
		Uncertain	--	--	--	32,552	55,554	36,719
Bassett Channel	CS-234	Fixed	.0806	.0595	.1018	6,557	55,554	8,277
		Uncertain	--	--	--	6,557	55,554	8,333
St. Clair Cutoff	CS-236	Fixed	.5403	.4921	.5884	43,939	55,554	47,855
		Uncertain	--	--	--	43,939	55,554	48,691
St. Clair Flats	CS-238	Fixed	.3791	.3204	.4379	30,831	55,554	35,613
		Uncertain	--	--	--	30,831	55,554	35,998
DETROIT RIVER								
Peche Island North	CS-003	Fixed	0.7350	0.7152	0.7549	139,186	55,554	142,951
Peche Island South	CS-008	Fixed	.2650	.2451	.2848	50,172	55,554	53,932
Scott Middle Ground	CS-015	Fixed	.3085	.2920	.3248	58,408	55,554	61,510
Fleming Channel	CS-029	Fixed	.6916	.6752	.7080	130,950	55,554	134,061
American Grassy Island	CS-100	Fixed	.2613	.2461	.2765	49,563	55,554	52,450
Fighting Island	CS-101	Fixed	.5227	.5101	.5353	99,143	55,554	101,532
Canadian Grassy Island	CS-102	Fixed	.2160	.2060	.2259	40,964	55,554	42,854
Trenton Channel	CS-120	Fixed	.2216	.2005	.2426	42,022	55,554	46,019
Grosse Ile-Stony Island	CS-121	Fixed	.0584	.0337	.0833	11,085	55,554	15,790
Upper Livingstone	CS-122	Fixed	.2602	.2400	.2803	49,347	55,554	53,169
Upper Amherstburg	CS-123	Fixed	.4598	.4360	.4836	87,216	55,554	91,730
Amherstburg Gap	CS-143	Fixed	.2410	.2013	.2806	21,015	55,554	24,474
		Uncertain	--	--	--	21,015	55,554	24,679
Lower Amherstburg	CS-165	Fixed	.7591	.7194	.7987	66,201	55,554	69,663
		Uncertain	--	--	--	66,201	55,554	71,114
Livingstone Gap East	CS-142	Fixed	.5982	.4090	.7879	12,572	55,554	16,557
		Uncertain	--	--	--	12,573	55,554	17,311
Bois Blanc Island-Dike	CS-164	Fixed	.4018	.2121	.5910	8,443	55,554	12,419
		Uncertain	--	--	--	8,443	55,554	12,833

Table 3. Summary of proportions and flows on selected branches of St. Clair and Detroit Rivers within the Great Lakes Waterway from Monte Carlo simulation—*Continued*

Branch	Cross section	Flow variability	Flow proportion			Flow (cubic feet per second)		
			Average	95% confidence limit		Average	95% confidence limit	
				Lower	Upper		Lower	Upper
DETROIT RIVER—*Continued*								
Livingstone Gap West	CS-141	Fixed	0.3560	0.2813	0.4309	22,043	55,554	26,680
		Uncertain	--	--	--	22,043	55,554	27,288
Lower Livingstone	CS-163	Fixed	.6440	.5691	.7187	39,876	55,554	44,502
		Uncertain	--	--	--	39,876	55,554	46,151
Sugar Island West	CS-161	Fixed	.6972	.6565	.7371	23,096	55,554	24,420
		Uncertain	--	--	--	23,115	55,554	28,031
Sugar Island East	CS-162	Fixed	.3028	.2629	.3435	10,032	55,554	11,380
		Uncertain	--	--	--	10,013	55,554	11,992

Table 4. Monte Carlo estimates of the ratios of variances of uncertain flows to fixed flows for specified flows at the headwaters of St. Clair River within the Great Lakes Waterway

Branch	Cross section	Ratio of the variance of flows (cubic feet per second)						
		170,000	180,000	190,000	200,000	210,000	220,000	230,000
South Channel	CS-232	1.0124	1.0115	1.0105	1.0105	1.0091	1.0087	1.0079
North Channel	CS-240	1.0263	1.0274	1.0282	1.0286	1.0301	1.0306	1.0320
Middle Channel	CS-242	1.0029	1.0029	1.0027	1.0027	1.0026	1.0026	1.0026
Bassett Channel	CS-234	1.0557	1.0518	1.0486	1.0489	1.0510	1.0568	1.0650
St. Clair Cutoff	CS-236	1.4766	1.4347	1.4108	1.4137	1.4313	1.4796	1.5471
St. Clair Flats	CS-238	1.1579	1.1459	1.1372	1.1373	1.1448	1.1588	1.1811
Amherstburg Gap	CS-143	1.1031	1.1017	1.1016	1.1015	1.1037	1.1088	1.1153
Lower Amherstburg	CS-165	2.0065	1.9881	1.9857	2.0039	2.0284	2.0677	2.1286
Livingstone Gap East	CS-142	1.3087	1.3048	1.3042	1.3068	1.3053	1.3066	1.3067
Bois Blanc Island-Dike	CS-164	1.1402	1.1410	1.1422	1.1403	1.1423	1.1425	1.1429
Livingstone Gap West	CS-141	1.2286	1.2259	1.2265	1.2297	1.2371	1.2450	1.2603
Lower Livingstone	CS-163	1.7478	1.7373	1.7365	1.7448	1.7623	1.7973	1.8475
Sugar Island West	CS-161	13.9841	13.1801	12.4723	11.7618	11.0001	10.2285	9.4250
Sugar Island East	CS-162	2.3994	2.1421	1.9227	1.7263	1.5558	1.4119	1.2876

Regression Estimators of Variance Ratios

Monte Carlo analysis is a computationally intensive application that often requires specialized software. To provide a system of equations that can be programmed easily into a spreadsheet, regression equations were developed to estimate variance ratios (VarRatio) shown in table 4 as a function of flow specified at the headwaters of St. Clair River. The polynomial equations are of maximum degree two (quadratic) as

$$VarRatio = \beta_0 + \beta_1 \cdot Q_{scr} + \beta_2 \cdot Q_{scr}^2 + \varepsilon \qquad (12)$$

where Q_{scr} is the specified flow at the headwaters of St. Clair River in cubic feet per second. Estimated parameters and associated statistics of the model fit are included in table 5. Comparison of results obtain by

Table 5. Regression coefficients for estimation of variance ratios in reaches with uncertain flows for St. Clair and Detroit Rivers within the Great Lakes Waterway

[--, no data; βs are defined in equation 12 as VarRatio = $\beta_0 + \beta_1 \cdot Q_{scr} + \beta_2 \cdot Q_{scr}^2 + \varepsilon$]

River and Branch	Cross section	β_0	β_1	β_2	Standard error of the estimate	Coefficient of determination (R^2)
St. Clair River						
South Channel	CS-232	1.025	-7.321×10^{-8}	--	0.00025	0.9766
North Channel	CS-240	1.011	9.071×10^{-8}	--	.00025	.9841
Middle Channel	CS-242	1.003	--	--	.00011	--
Bassett Channel	CS-234	1.540	-5.051×10^{-6}	1.299×10^{-11}	.00042	.9949
St. Clair Cutoff	CS-236	5.646	-4.349×10^{-5}	1.116×10^{-10}	.00184	.9988
St. Clair Flats	CS-238	2.492	-1.391×10^{-5}	3.569×10^{-11}	.00078	.9979
Detroit River						
Amherstburg Gap	CS-143	1.398	-3.149×10^{-6}	8.345×10^{-12}	.00064	.9874
Lower Amherstburg	CS-165	4.536	-2.738×10^{-5}	7.352×10^{-11}	.00228	.9984
Livingstone Gap East	CS-142	1.306	--	--	.00152	--
Bois Blanc Island-Dike	CS-164	1.1419	--	--	.00111	--
Livingstone Gap West	CS-141	1.769	-5.910×10^{-6}	1.606×10^{-11}	.00078	.9968
Lower Livingstone	CS-163	3.811	-2.226×10^{-5}	5.963×10^{-11}	.00266	.9965
Sugar Island West	CS-161	26.76	-7.519×10^{-5}	--	.03285	.9996
Sugar Island East	CS-162	10.62	-7.055×10^{-5}	1.303×10^{-10}	.00305	.9999

use of Monte Carlo simulation and results obtained by use of the regression equation estimates of the variance ratio and applied to developing parametric confidence interval (table 6) are in close agreement. Thus, the regression equations can be used to estimate variance ratios needed to develop parametric confidence intervals for branches with uncertain flows without the need for additional Monte Carlo simulations.

Finally, a numerical example will be provided to illustrate the computations involved in estimating variance ratios and parametric confidence intervals for flow in reaches with uncertain flow. Consider flow on Detroit River at Bois Blanc Island Reach, Lower Amherstburg Branch (cross section CS-165). In Detroit River, flow downstream of the Grosse Ile-Stony Island-Dike Reach (including cross sections CS-120, CS-121, CS-122, and CS-123) becomes uncertain because only one of the four branches (Upper Amherstburg, cross section CS-123) contributes to flow in Bois Blanc Island Reach. Flow in Bois Blanc Island Reach branches into the Amherstburg Gap (CS-143) and Lower Amherstburg (CS-165).

For a specified flow of 180,000 ft³/s at the headwaters of St. Clair River and average inflows otherwise, the total flow in the Grosse Ile-Stony Island-Dike reach is 189,670 ft³/s. By use of the equations in table 1, the estimated proportion of flow at cross-section CS-123 is 0.4598, with a resulting flow of 87,210 ft³/s. The estimated proportion of flow in cross-section CS-165 is a constant (1 - 0.24095) of the flow in cross-section CS-123. Thus, the expected flow in cross-section CS-165 for the specified flow is 66,223 ft³/s. The 95-percent confidence interval, ci_{CS-165} of this flow, considering only the uncertainty in the flow proportion is

$$ci_{CS-165} = \begin{cases} \hat{q}_{CS-123}(\hat{p}_{CS-165} \pm t_{0.05,5} \cdot \hat{\sigma}_{CS-165}) \\ [62{,}735 \quad 69{,}657\ ]\ ft^3 s^{-1} \end{cases}.$$

To describe the total uncertainty, the variance ratio is estimated by use of parameters in table 5 as

$$\hat{V}arRatio_{CS-165} = \begin{cases} 4.536 - 2.738 \cdot 10^{-5} \cdot 180{,}000 \\ \quad + 7.352 \cdot 10^{-11} \cdot 180{,}000^2 \\ 1.9896 \end{cases},$$

and applied to compute the 95-percent confidence interval of the flow in cross-section CS-165 as

$$CI_{CS-165} = \begin{cases} \hat{q}_{CS-123}\left(\hat{p}_{CS-165} \pm t_{0.05,5} \cdot \sigma_{CS-165}\sqrt{\hat{V}arRatio_{CS-165}}\right) \\ [61,314 \qquad 71,078] \cdot ft^3 s^{-1} \end{cases} .$$

Table 6. Parametric estimates of flows and proportions of selected branches of St. Clair and Detroit Rivers within the Great Lakes Waterway computed by use of estimated variance ratios

[%, percent; --, no data]

Branch	Cross section	Flow variability	Flow proportion			Flow (cubic feet per second)		
			Expected	95% confidence limit		Expected	95% confidence limit	
				Lower	Upper		Lower	Upper
ST. CLAIR RIVER								
Stag Island East	CS-208	Fixed	0.3225	0.3078	0.3372	58,212	55,556	60,868
Stag Island West	CS-210	Fixed	.6775	.6628	.6922	122,277	119,635	124,933
Fawn Island West	CS-216	Fixed	.8252	.8147	.8357	149,437	147,533	151,341
Fawn Island East	CS-218	Fixed	.1748	.1643	.1853	31,649	29,745	33,553
Chenal Ecarte	CS-222	Fixed	.0381	.0306	.0457	6,908	5,547	8,269
Lower St. Clair River	CS-230	Fixed	.9619	.9543	.9694	174,189	172,817	175,539
South Channel	CS-232	Fixed	.4669	.4395	.4943	81,325	76,552	86,098
		Uncertain	--	--	--	--	76,524	86,126
North Channel	CS-240	Fixed	.3462	.3256	.3669	60,306	56,709	63,903
		Uncertain	--	--	--	--	56,660	63,952
Middle Channel	CS-242	Fixed	.1869	.1637	.2101	32,558	28,513	36,602
		Uncertain	--	--	--	--	28,508	36,607
Bassett Channel	CS-234	Fixed	.0806	.0594	.1018	6,553	4,829	8,277
		Uncertain	--	--	--	--	4,785	8,321
St. Clair Cutoff	CS-236	Fixed	.5403	.4918	.5887	43,936	39,998	47,874
		Uncertain	--	--	--	--	39,203	48,669
St. Clair Flats	CS-238	Fixed	.3792	.3202	.4381	30,836	26,040	35,632
		Uncertain	--	--	--	--	25,708	35,967
DETROIT RIVER								
Peche Island North	CS-003	Fixed	0.7350	0.7152	0.7549	139,184	135,424	142,944
Peche Island South	CS-008	Fixed	.2650	.2451	.2848	50,174	46,414	53,934
Scott Middle Ground	CS-015	Fixed	.3085	.2920	.3249	58,409	55,291	61,527
Fleming Channel	CS-029	Fixed	.6915	.6751	.7080	130,949	127,408	134,067
American Grassy Island	CS-100	Fixed	.2613	.2461	.2765	49,561	46,670	52,452
Fighting Island Channel	CS-101	Fixed	.5227	.5101	.5353	99,144	96,748	101,529
Canadian Grassy Island	CS-102	Fixed	.2160	.2060	.2259	40,965	39,078	42,852
Trenton Channel	CS-120	Fixed	.2216	.2005	.2426	42,023	38,032	46,014
Grosse Ile-Stony Island	CS-121	Fixed	.0585	.0336	.0833	11,090	6,382	15,797
Upper Livingstone	CS-122	Fixed	.2602	.2399	.2805	49,347	45,501	53,194
Upper Amherstburg	CS-123	Fixed	.4598	.4369	.4827	87,210	82,861	91,558

Table 6. Parametric estimates of flows and proportions of selected branches of St. Clair and Detroit Rivers within the Great Lakes Waterway computed by use of estimated variance ratios—*Continued*

Branch	Cross section	Flow variability	Flow proportion			Flow (cubic feet per second)		
			Expected	95% confidence limit		Expected	95% confidence limit	
				Lower	Upper		Lower	Upper
DETROIT RIVER—*Continued*								
Amherstburg Gap.............	CS-143	Fixed	0.2410	0.2013	0.2806	21,013	17,552	24,474
		Uncertain	--	--	--	--	17,381	24,646
Lower Amherstburg...........	CS-165	Fixed	.7591	.7194	.7987	66,197	62,736	69,657
		Uncertain	--	--	--	--	61,315	71,078
Livingstone Gap East........	CS-142	Fixed	.5982	.4084	.7880	12,571	8,583	16,558
		Uncertain	--	--	--	--	8,013	17,128
Bois Blanc Island-Dike.....	CS-164	Fixed	.4018	.2129	.5907	8,443	4,474	12,411
		Uncertain	--	--	--	--	4,181	12,704
Livingstone Gap West.......	CS-141	Fixed	.3561	.2812	.4310	22,047	17,408	26,685
		Uncertain	--	--	--	--	16,912	27,181
Lower Livingstone	CS-163	Fixed	.6439	.5690	.7188	39,871	35,233	44,510
		Uncertain	--	--	--	--	33,760	45,983
Sugar Island West.............	CS-161	Fixed	.6972	.6587	.7358	23,104	21,827	24,380
		Uncertain	--	--	--	--	18,461	27,747
Sugar Island East..............	CS-162	Fixed	.3028	.2642	.3413	10,032	8,756	11,309
		Uncertain	--	--	--	--	8,164	11,901

FLOW-DURATION ANALYSIS

Continuous flow data on St. Clair and Detroit Rivers are compiled by Federal agencies in the United States and Canada. Representatives from these agencies meet periodically, as the Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data, to review and finalize these flow data. In 1988, the committee published coordinated monthly average flow data from 1900 to 1986 (Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data, 1988). On April 18, 2001, these data were retrieved by ftp (file transfer protocol) from the universal resource locator (URL) ftp.glerl.noaa.gov/ publications/... tech_reports/glerl-083/Spreadsheets/ Channels.xls (Please note that the "..." above indicates a continuation of the URL on the next line of text, but is not part of the URL.)

A 360-month period from January 1960 to December 1989 was selected from this data repository to describe recent flow characteristics on St. Clair and Detroit Rivers. A time-series plot (fig. 10) indicates that

St. Clair and Detroit Rivers flows show a similar seasonal pattern, with Detroit River flows consistently higher than St. Clair River flows. Also, St. Clair River and Detroit River have similar monthly flow-duration characteristics (fig. 11). The flow-duration plot was created by arranging monthly average flows in the order of decreasing magnitude and plotting them and the percent of months in which the flow was equaled or exceeded.

Flows with 100 equally spaced percent exceedances ranging from 0.5 and 99.5 percent exceedance were interpolated from flow-duration curves for St. Clair and Detroit Rivers. These interpolated flows were assigned as the flow boundary at the headwaters of the respective rivers and used with the flow proportion equations to compute corresponding flows for the selected branches of St. Clair and Detroit Rivers. Estimates of flow duration characteristics at flow exceedances of less than 10 percent or greater than 85 percent are based on an extrapolation of the equations for computing flow proportions. These extrapolated estimates are more uncertain than flow exceedance estimates within the range of 10 to 85 percent. The computed flows were used to develop flow-duration curves for the branches (figs. 12–14). Tributaries inflows between the headwaters and the branches were held constant. No inflows were used for Clinton, Thames, or Sydenham Rivers or direct inflow into Lake St. Clair because these flow were accounted for in the monthly average flows for Detroit River.

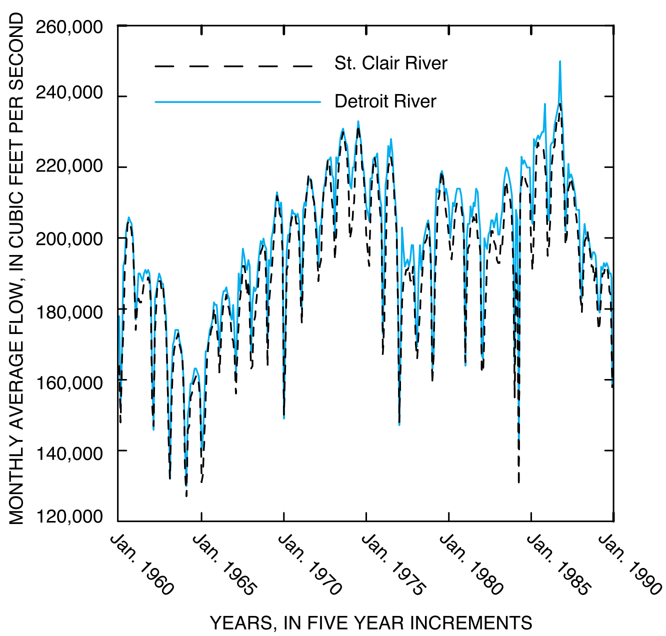

Figure 10. Time-series plot of monthly average flows for St. Clair and Detroit Rivers within the Great Lakes Waterway.

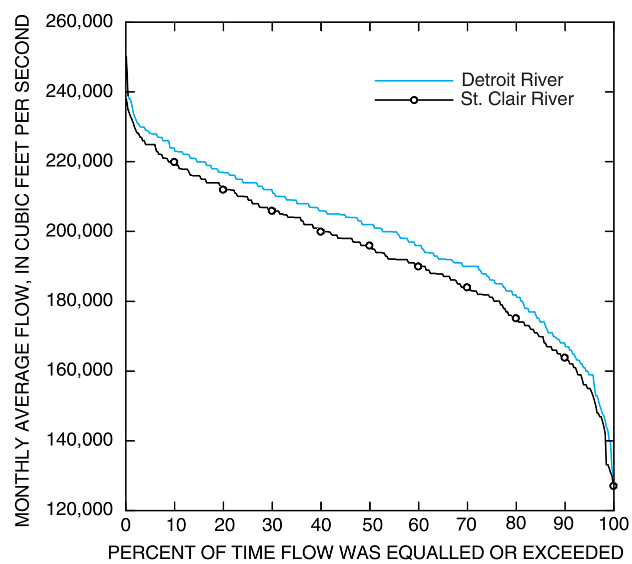

Figure 11. Monthly flow-duration characteristics for St. Clair and Detroit Rivers within the Great Lakes Waterway.

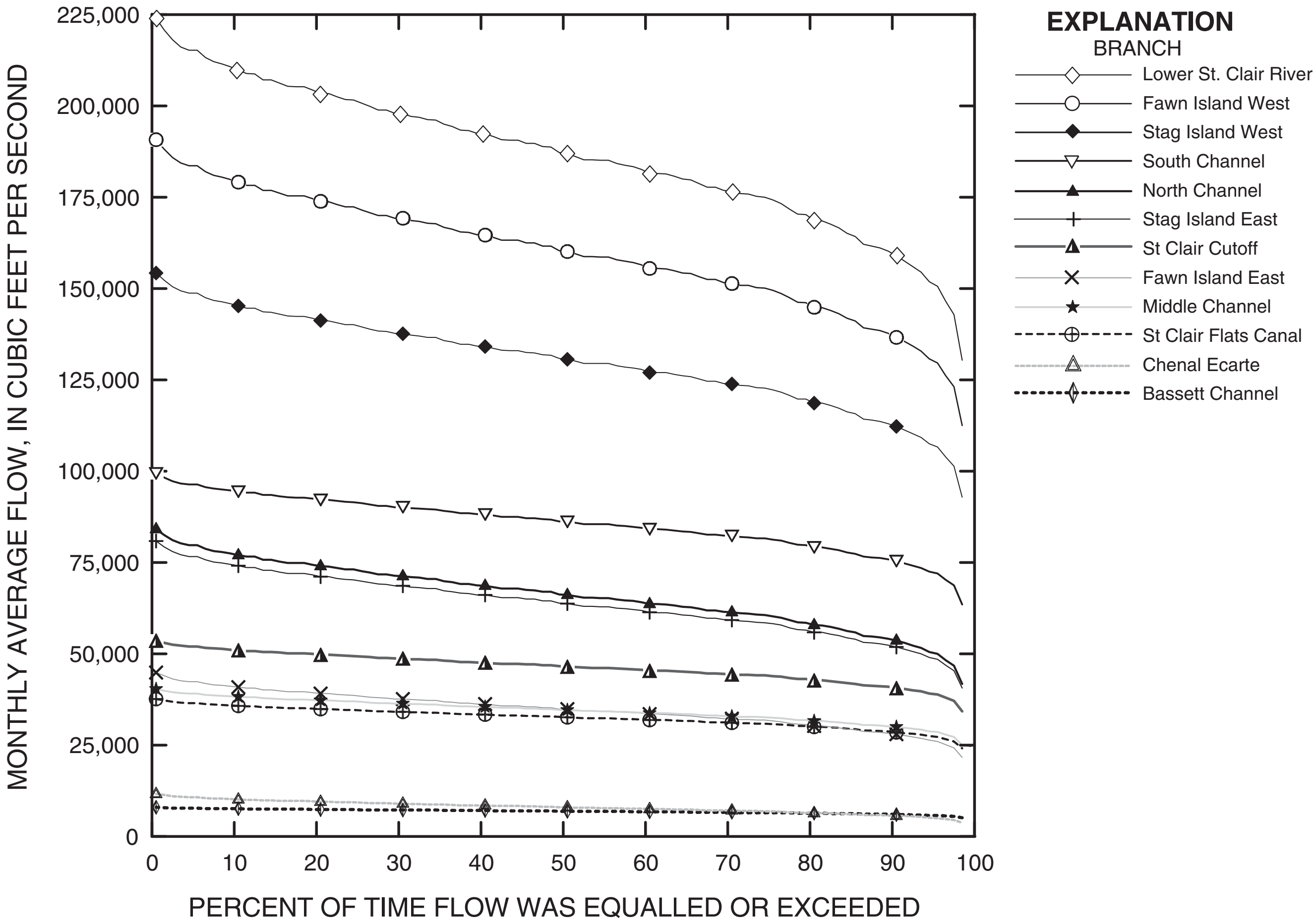

Figure 12. Estimated monthly flow-duration characteristics for selected branches of St. Clair River within the Great Lakes Waterway.

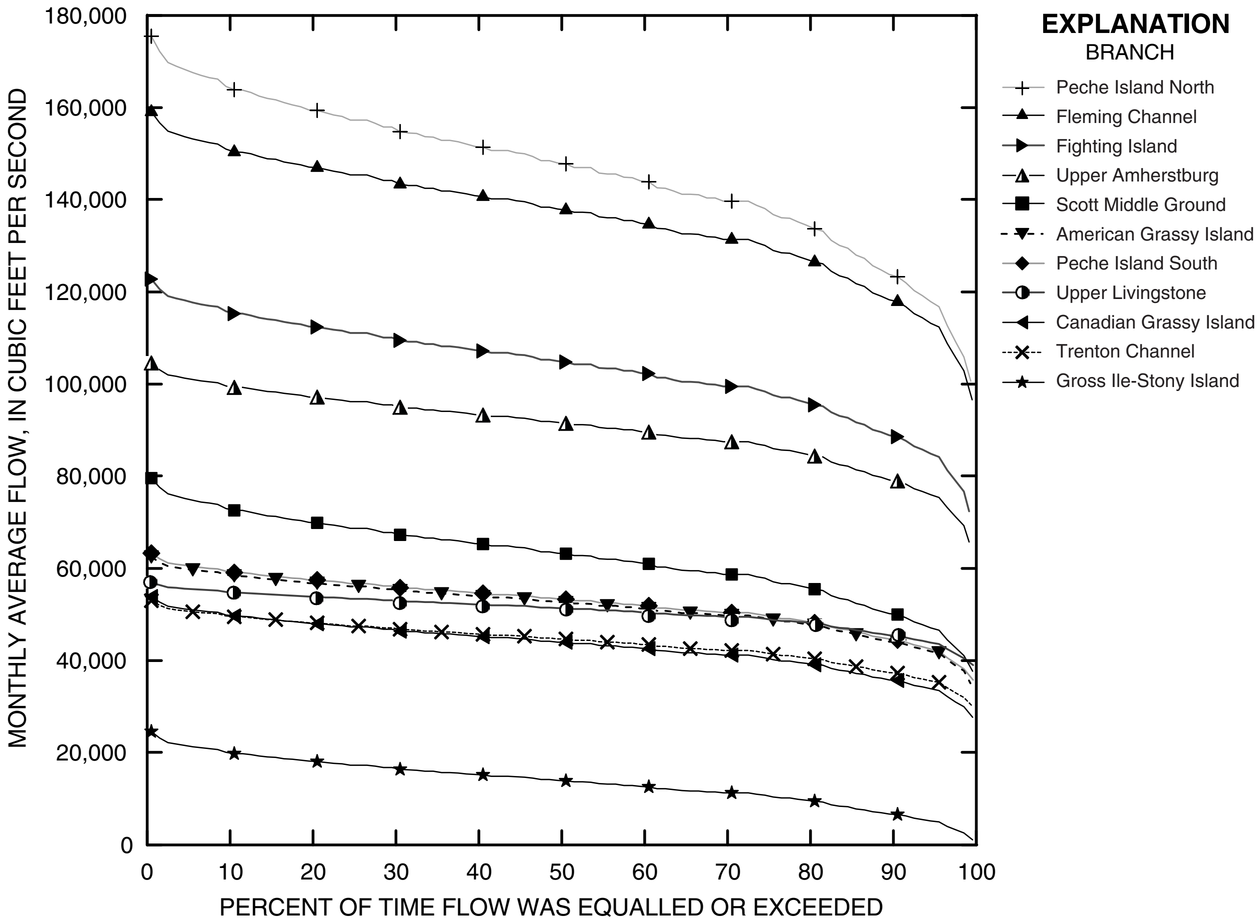

Figure 13. Estimated monthly flow-duration characteristics for selected branches of upper Detroit River within the Great Lakes Waterway.

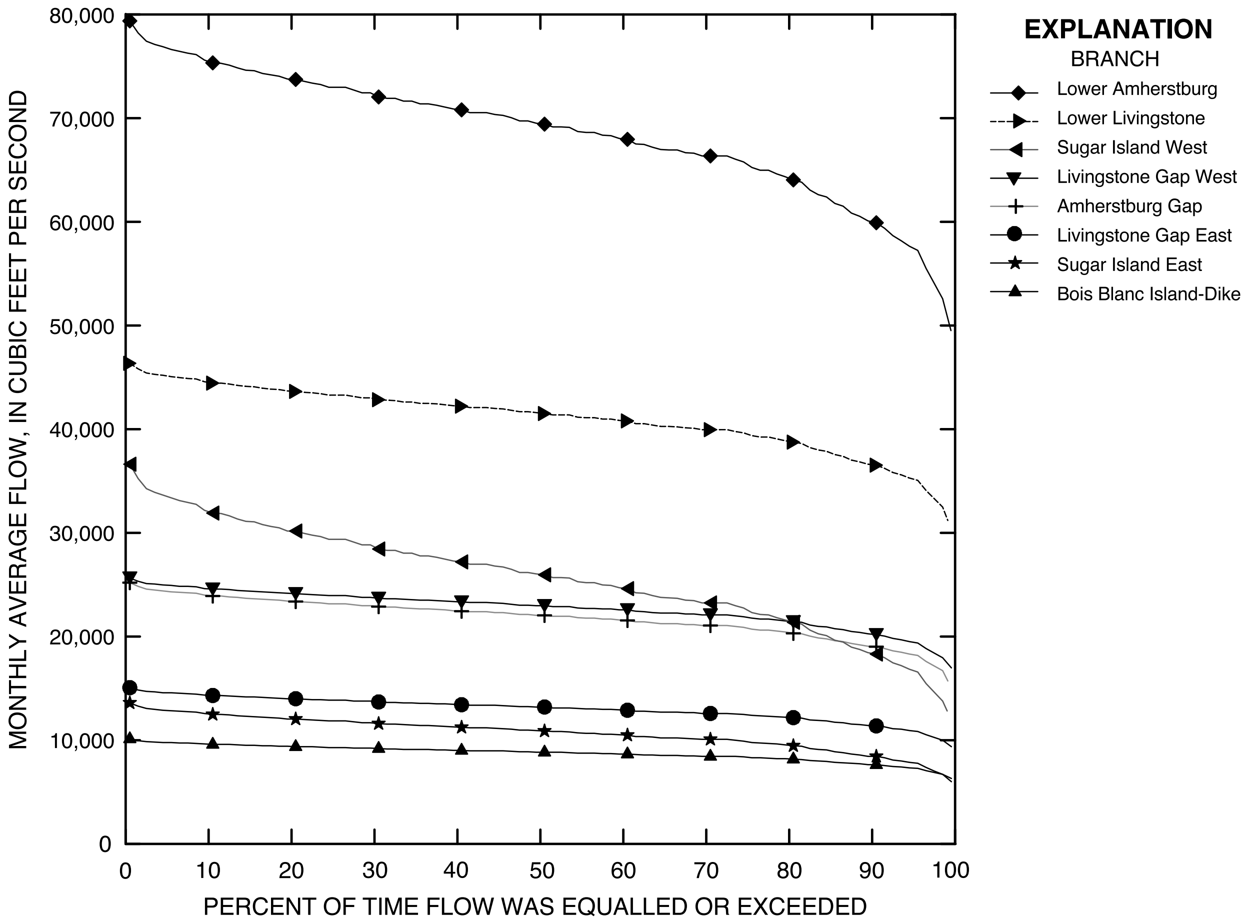

Figure 14. Estimated monthly flow-duration characteristics for selected branches of lower Detroit River within the Great Lakes Waterway.

SUMMARY AND CONCLUSIONS

The Michigan Department of Environmental Quality (MDEQ) Source Water Assessment Program (SWAP) and the Detroit Water and Sewerage Department (DWSD) are assessing the susceptibility of public water intakes to contamination on the St. Clair and Detroit Rivers within the Great Lakes Waterway. The U.S. Geological Survey and Detroit District of the U.S. Army Corps of Engineers are assisting the MDEQ and DWSD in this effort by developing an integrated hydrodynamic model of St. Clair River, Lake St. Clair, and Detroit River. This study develops flow distribution and flow duration information needed to calibrate this hydrodynamic model.

In some reaches of the St. Clair and Detroit Rivers, islands and dikes split the flow into 2 to 4 branches (channels). Even when the flow in the reach is known, however, the (proportion of) flow that is distributed to individual branches is uncertain. This study uses 533 ADCP (acoustic Doppler current profiler) flow measurements obtained during ice-free conditions from 1996 and 2000 to develop simple linear regression equations for estimating these flow proportions as a function of the total flow in the reach. These equations provide a method for estimating flow proportions in 5 reaches on St. Clair River containing 12 branches, and 8 reaches on Detroit River containing 19 branches. Flow estimates provided by the equations can be compared with measured flows to quality assurance measurement data.

Regression equations are developed with a continuity constraint so that estimated flow proportions in all branches within a reach sum to one. Because of this constraint, flow proportions could only be estimated independently in one fewer than the total number of branches within a reach. For example, two equations are developed for estimating flow proportions in a three-branch reach; the flow proportion in the remaining branch is computed by subtracting the estimated proportions from one. Results of the regression analysis indicate that the flow proportion in a branch is a function of the total flow within the reach in 9 of 12 branches on St. Clair River, and in 9 of 19 branches on Detroit River. In other branches, flow proportions are constant.

The uncertainties of the flow proportions are used to infer the uncertainties of the corresponding flows. In three upstream reaches on St. Clair River and four upstream reaches on Detroit River, the total flow is considered fixed because it is constrained by continuity to equal to the sum of flow specified at the upstream flow boundaries. Boundaries where flow can be specified include the headwaters of St. Clair River, and at the mouths of selected tributaries including Black, Pine, and Belle Rivers on St. Clair River; Clinton, Thames, and Sydenham Rivers on Lake St. Clair; and River Rouge on Detroit River. In addition, a net inflow for Lake St. Clair can be specified. In reaches where the flow is fixed, the uncertainty of the flow proportions indicated by the regression equations can be used to determine the uncertainty of the corresponding flows directly.

In other reaches, flow is considered uncertain because the continuity constraint is not applicable. This can occur when the local reach does not receive flow from all the branches of an upstream reach, or it receives flow from some branches of more than one upstream reach. This lack of continuity allows uncertainty in the flow proportions of upstream reaches to cascade downstream and affect uncertainties of flow proportions in downstream reaches. Further, this cascade of uncertainty creates an estimate of flows in downstream branches that are a product of random proportions. The uncertainties of these products may not follow a *t*-distribution, even if the uncertainties of the individual proportions making up the products are *t*-distributed. If these uncertainties are not *t*-distributed, then the parametric confidence limits formed on the basis of the *t* distribution are not applicable.

Monte Carlo simulation analysis is used to assess the impact of this cascade of uncertainty on the magnitude and distribution of estimated flow proportions. Monte Carlo analysis uses random sampling, based on a *t*-distribution of uncertainties of flow proportions in reaches with fixed flows, to generate likely realizations of flow proportions in these upstream reaches. These realizations are used with boundary specifications to randomly generate flow proportions in downstream reaches, where flows may be uncertain.The impacts of the cascade on the magnitude of the uncertainty of downstream flow proportions are described by a variance ratio.The ratio is computed by dividing the variance of flow proportions in the downstream branch that includes the uncertainty of upstream flow proportions by the variance of flow proportions computed only by random sampling from a *t*-distribution describing the uncertainty of downstream flow proportions. All these variance ratios are greater than one and generally vary with flow magnitudes at the boundaries. The impacts of the cascade on the distribution of uncertainties was investigated by comparing nonparametric confidence limits computed by use of the simulated flow proportions with parametric confidence limits computed on the basis of the *t*-distribution. Results indicate that parametric confidence limits formed by the use of the *t*-distribution are in close agreement with the nonparametric limits.

To eliminate the need for Monte Carlo simulations in routine calculations, the variation of the variance ratios with flow specified at the boundaries is approximated by use of polynomial regression equations with maximum degree of two. The equations closely approximate the variation in variance ratios computed for a range of flows from 170,000 ft^3/s to 230,000 ft^3/s at the headwaters of St. Clair River. For reaches with uncertain flows, these polynomial equations for estimating variance ratios can be used with the regression equations for estimating flow proportions to determine the magnitude and uncertainty of flows.

The Coordinating Committee for Great Lakes Basic Hydraulic and Hydrologic Data, which includes scientists representing Federal agencies in the United States and Canada, coordinates flows on the Great Lakes. Monthly average flows from January 1960 to December 1989, the most recent period for which coordinated flows are published for St. Clair and Detroit Rivers, were used to determine monthly flow-duration characteristics of St. Clair and Detroit River. This flow duration information indicates the percent of time that monthly average flow of a specified magnitude is exceeded on the main branches of the rivers. This flow information on the main branches was combined the equations for computing flow proportions developed in this report to describe the flow duration characteristics of 12 branches on St. Clair River and 19 branches on Detroit River.

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APPENDIX A.
Selected Flow Measurements

Table A1. Selected flow measurements on St. Clair River near Stag Island, Ontario, Canada

[ft³/s, cubic feet per second]

Measurement date	Flow at CS-208 (ft^3/s)	Number of transects	Flow at CS-210 (ft^3/s)	Number of transects
May 14, 1996	65,800	2	128,000	2
June 11, 1996	74,200	2	140,000	2
July 9, 1996	72,100	3	147,000	2
May 6, 1997	75,500	2	151,000	2
June 10, 1997	73,700	3	145,000	3
August 5, 1997	76,900	2	146,000	3
September 9, 1997	73,900	3	147,000	3
October 7, 1997	73,100	3	139,000	3
July 22, 1998	70,600	2	139,000	4
August 10, 1998	70,900	3	143,000	2
September 22, 1998	67,800	3	137,000	2
October 27, 1998	64,200	2	133,000	3
June 15, 1999	61,900	3	130,000	3
July 21, 1999	60,900	2	127,000	2
August 26, 1999	58,900	2	122,000	2
September 23, 1999	53,500	3	117,000	2
November 4, 1999	58,300	2	124,000	2
Average	67,770		136,210	
Standard deviation	7,032		10,167	

Table A2. Selected flow measurements on St. Clair River near Fawn Island, Ontario, Canada

[ft^3/s, cubic feet per second]

Measurement date	Flow at CS-216 (ft^3/s)	Number of transects	Flow at CS-218 (ft^3/s)	Number of transects
May 14, 1996	155,000	2	34,600	2
June 11, 1996	174,000	3	38,200	2
July 9, 1996	170,000	4	38,700	3
May 6, 1997	180,000	3	40,500	2
June 10, 1997	177,000	2	40,000	2
August 5, 1997	182,000	2	44,400	2
September 9, 1997	182,000	2	39,600	3
October 7, 1997	181,000	2	41,700	2
July 22, 1998	169,000	3	38,000	2
August 10, 1998	172,000	3	39,500	2
September 22, 1998	169,000	2	38,200	2
October 27, 1998	163,000	3	35,500	3
June 15, 1999	164,000	3	35,400	2
July 21, 1999	156,000	2	33,000	2
August 26, 1999	145,000	3	31,200	2
September 23, 1999	145,000	3	28,500	2
November 4, 1999	152,000	3	31,100	2
June 29, 2000	140,000	2	30,500	2
October 5, 2000	145,000	2	30,500	2
Average	164,230		36,244	
Standard deviation	14,106		4,463	

Table A3. Selected flow measurements on St. Clair River near Walpole Island, Ontario, Canada

[ft^3/s, cubic feet per second]

Measurement date	Flow at CS-222 (ft^3/s)	Number of Transects	Measurement date	Flow at CS-230 (ft^3/s)	Number of transects
May 14, 1996	8,100	2	May 13, 1996	196,000	3
June 10, 1996	9,400	2	June 10, 1996	199,000	2
July 8, 1996	9,330	3	July 8, 1996	196,000	5
May 5, 1997	7,930	3	May 5, 1997	188,000	2
June 10, 1997	10,300	2	June 9, 1997	208,000	4
August 5, 1997	11,200	2	August 4, 1997	216,000	1
September 9, 1997	10,800	2	September 8, 1997	209,000	4
October 6, 1997	10,200	2	October 6, 1997	216,000	2
July 22, 1998	7,230	3	July 21, 1998	194,000	3
August 10, 1998	9,400	1	August 10, 1998	196,000	4
September 22, 1998	9,900	2	September 21, 1998	198,000	5
October 27, 1998	8,750	2	October 26, 1998	185,000	4
June 15, 1999	8,600	2	June 14, 1999	202,000	2
July 21, 1999	8,350	2	July 20, 1999	184,000	2
August 26, 1999	7,250	2	August 25, 1999	173,000	2
November 4, 1999	7,400	2	November 3, 1999	190,000	2
June 29, 2000	5,800	2	June 27, 2000	167,000	2
Average	8,809			195,170	
Standard deviation	1,431			13,571	

Table A4. Selected flow measurements on St. Clair River near Dickinson-Harsens Islands, Michigan

[ft^3/s, cubic feet per second]

Measurement date	Flow at CS-232 (ft^3/s)	Number of transects	Flow at CS-240 (ft^3/s)	Number of transects	Flow at CS-242 (ft^3/s)	Number of transects
May 13, 1996	85,700	2	70,300	2	37,400	3
June 10, 1996	90,300	2	71,700	2	30,100	3
July 8, 1996	86,100	2	66,400	2	34,100	4
May 5, 1997	83,600	2	59,500	2	32,100	2
June 9, 1997	89,100	2	76,600	4	38,500	2
August 4, 1997	97,300	3	82,200	1	39,600	2
September 8, 1997	96,000	3	80,200	2	38,800	2
October 6, 1997	96,900	2	75,300	3	38,800	2
July 21, 1998	86,600	2	67,000	4	37,100	2
August 10, 1998	89,800	3	66,600	3	36,100	2
September 21, 1998	87,800	3	68,600	2	37,000	2
October 26, 1998	85,100	2	64,600	2	34,500	3
June 14, 1999	90,200	3	62,500	3	32,200	2
July 20, 1999	82,300	2	62,100	2	34,300	2
August 25, 1999	77,300	2	59,600	2	34,600	2
June 27, 2000	79,700	2	60,700	2	30,600	2
Average	87,727		68,353		35,339	
Standard deviation	5,775		7,174		3,017	

Table A5. Selected flow measurements on St. Clair River at Bassett-Seaway Island, Ontario, Canada

[ft^3/s, cubic feet per second]

Measurement date	Flow at CS-234 (ft^3/s)	Number of transects	Flow at CS-236 (ft^3/s)	Number of transects	Flow at CS-238 (ft^3/s)	Number of transects
May 13, 1996	7,500	2	47,000	2	29,500	2
June 10, 1996	6,970	3	46,100	2	33,600	2
July 8, 1996	7,130	3	46,800	3	36,000	3
May 5, 1997	4,200	2	42,700	3	37,400	3
June 9, 1997	7,500	2	49,600	2	31,800	2
August 4, 1997	8,500	2	55,000	2	34,200	2
September 8, 1997	8,200	2	51,800	2	36,600	2
October 6, 1997	7,950	2	51,600	2	35,900	2
July 21, 1998	7,650	2	42,700	3	35,800	3
August 10, 1998	6,800	2	46,700	3	33,300	3
September 21, 1998	7,250	2	47,300	2	32,000	2
October 26, 1998	6,800	2	43,300	3	32,600	3
June 14, 1999	7,600	2	45,100	2	27,300	2
July 20, 1999	6,450	2	46,700	3	30,500	3
August 25, 1999	5,500	2	41,600	2	32,600	2
November 3, 1999	6,500	2	51,800	2	30,200	2
June 27, 2000	6,350	2	40,800	2	29,000	2
Average	6,991		46,842		32,834	
Standard deviation	1,034		4,026		2,925	

Table A6. Selected flow measurements on Detroit River near Peche Island, Ontario, Canada

[ft^3/s, cubic feet per second]

Measurement date	Flow at CS-3 (ft^3/s)	Number of Transects	Flow at CS-8 (ft^3/s)	Number of transects
May 16, 1996	156,000	2	58,900	4
June 24, 1996	165,000	3	62,000	2
July 24, 1996	167,000	3	54,400	3
April 30, 1997	153,000	2	58,400	3
May 22, 1997	181,000	3	66,500	3
June 24, 1997	177,000	2	63,600	2
July 24, 1997	169,000	3	55,600	3
August 20, 1997	179,000	2	61,500	4
September 25, 1997	177,000	3	66,000	3
October 24, 1997	164,000	3	61,700	4
July 20, 1998	159,000	4	56,600	3
August 3, 1998	153,000	3	54,000	2
September 10, 1998	153,000	3	57,500	2
October 6, 1998	124,000	3	45,400	3
May 26, 1999	148,000	2	51,600	3
July 6, 1999	144,000	2	48,800	3
September 15, 1999	144,000	3	54,100	2
October 6, 1999	152,000	2	54,300	2
November 17, 1999	138,000	3	51,800	3
Average	158,072		56,972	
Standard deviation	15,037		5,680	

Table A7. Selected flow measurements on Detroit River near Belle Isle, Michigan

[ft^3/s, cubic feet per second]

Measurement date	Flow at CS-15 (ft^3/s)	Number of Transects	Flow at CS-29 (ft^3/s)	Number of transects
May 16, 1996	69,600	4	143,000	2
June 24, 1996	74,900	4	159,000	2
July 24, 1996	69,900	2	149,000	4
April 30, 1997	73,900	4	148,000	2
May 22, 1997	83,100	3	161,000	3
June 24, 1997	76,100	4	154,000	3
July 24, 1997	72,800	4	155,000	4
August 20, 1997	74,100	4	155,000	4
September 25, 1997	75,600	3	149,000	4
October 24, 1997	76,400	3	154,000	3
July 20, 1998	71,000	3	148,000	2
August 3, 1998	64,800	3	141,000	3
September 10, 1998	67,600	2	143,000	3
October 6, 1998	59,500	2	131,000	3
May 26, 1999	64,400	3	134,000	5
July 6, 1999	59,400	3	138,000	3
September 15, 1999	60,500	2	133,000	3
October 6, 1999	62,800	2	142,000	2
November 17, 1999	58,600	2	132,000	3
Average	69,189		145,710	
Standard deviation	7,062		9,287	

Table A8. Selected flow measurements on Detroit River near Fighting Island and Grassy Island, Ontario, Canada, and Grassy Island, Michigan

[ft^3/s, cubic feet per second]

Measurement date	Flow at CS-100 (ft^3/s)	Number of transects	Flow at CS-101 (ft^3/s)	Number of transects	Flow at CS-102 (ft^3/s)	Number of transects
May 20, 1996	55,930	4	111,000	2	45,400	3
June 25, 1996	60,570	3	121,000	3	51,400	4
July 23, 1996	59,430	3	116,600	2	49,000	2
May 2, 1997	65,600	2	120,700	2	53,600	3
May 21, 1997	69,630	3	127,400	3	54,700	2
June 25, 1997	57,000	2	109,700	3	47,700	3
July 23, 1997	60,400	3	119,000	4	53,200	2
August 19, 1997	56,300	3	112,900	3	47,200	3
September 24, 1997	60,330	3	122,700	3	52,400	2
October 23, 1997	58,800	2	116,600	3	51,400	2
August 4, 1998	52,000	2	107,600	3	46,200	3
September 9, 1998	52,970	3	111,200	3	46,500	2
October 5, 1998	53,550	2	108,000	2	46,800	2
May 27, 1999	50,300	2	98,400	2	40,800	2
September 16, 1999	1999	2	97,633	3	41,600	2
October 7, 1999	48,400	2	98,550	2	40,700	2
November 18, 1999	45,350	2	92,150	2	37,600	2
May 23-24, 2000	46,525	2	88,825	2	36,600	2
Average	55,527		109,990		46,804	
Standard deviation	6,732		11,043		5,534	

Table A9. Selected flow measurements on Detroit River near Grosse Ile and Stony Island, Michigan

[ft^3/s, cubic feet per second]

Measurement date	Flow at CS-120 (ft^3/s)	Number of transects	Flow at CS-121 (ft^3/s)	Number of transects	Flow at CS-122 (ft^3/s)	Number of transects	Flow at CS-123 (ft3/s)
May 20, 1996	47,100	2	19,500	3	49,200	4	92,500
June 25, 1996	53,100	2	22,300	3	54,300	2	101,000
July 23, 1996	50,900	2	17,900	2	52,900	3	95,100
May 21, 1997	57,900	2	26,600	3	55,900	2	102,400
June 25, 1997	49,700	2	23,100	2	52,500	2	94,000
July 23, 1997	52,700	2	19,300	2	53,500	2	97,400
August 19, 1997	47,900	3	19,000	2	55,500	3	98,600
September 24, 1997	52,800	3	20,600	3	58,700	3	107,000
October 23, 1997	47,600	4	19,200	3	54,300	3	98,200
August 4, 1998	44,300	2	16,600	3	54,200	2	94,200
September 9, 1998	44,200	2	14,800	2	57,100	2	95,900
October 5, 1998	43,900	2	14,000	2	53,000	2	93,500
May 27, 1999	43,000	2	14,200	2	49,700	2	86,700
July 7, 1999	41,800	2	8,800	2	49,300	2	91,800
October 7, 1999	40,000	2	9,600	2	53,200	2	94,100
November 18, 1999	39,100	2	10,000	2	47,000	2	85,100
May 23-25, 2000	39,700	7	7,750	2	46,000	4	82,400
Average	46,787		16,649		52,703		94,692
Standard deviation	5,472		5,412		3,471		6,206

Table A10. Selected flow measurements on Detroit River near Bois Blanc Island, Ontario, Canada

[ft^3/s, cubic feet per second]

Measurement date	Flow at CS-143 (ft^3/s)	Number of Transects	Flow at CS-165 (ft^3/s)	Number of transects
June 25, 1996	23,200	3	76,500	2
July 23, 1996	22,000	3	72,900	3
May 2, 1997	19,700	3	59,500	3
May 21, 1997	28,400	2	74,600	3
June 25, 1997	26,700	2	71,900	3
July 23, 1997	21,800	3	71,600	2
August 19, 1997	21,400	3	83,100	3
September 24, 1997	24,200	2	72,500	3
October 23, 1997	22,800	2	71,500	2
August 4, 1998	23,400	2	74,100	3
September 9, 1998	23,700	3	77,400	3
October 5, 1998	21,800	2	71,800	2
May 28, 1999	20,100	2	60,400	2
October 7, 1999	19,200	2	69,600	2
November 18, 1999	19,900	2	55,400	2
Average	22,365		70,571	
Standard deviation	2,567		7,111	

Table A11. Selected flow measurements on Detroit River near Bois Blanc Island and Livingstone Channel, Ontario, Canada

[ft^3/s, cubic feet per second]

Measurement date	Flow at CS-142 (ft^3/s)	Number of Transects	Flow at CS-164 (ft^3/s)	Number of transects
July 23, 1996	14,400	2	9,880	4
May 2, 1997	7,050	2	9,750	2
May 21, 1997	16,100	2	11,100	2
June 25, 1997	13,000	3	12,100	3
July 23, 1997	11,900	2	7,800	3
August 19, 1997	11,700	3	7,770	3
September 24, 1997	14,700	2	9,770	3
October 23, 1997	11,000	2	10,000	3
August 4, 1998	13,200	2	8,250	2
September 9, 1998	12,100	1	8,100	2
October 5, 1998	12,500	3	8,500	2
May 28, 1999	12,200	3	8,450	2
July 7, 1999	19,700	2	4,530	3
September 16, 1999	12,700	3	[1]5,850	2
October 7, 1999	12,300	2	6,200	2
November 18, 1999	14,600	2	6,550	2
Average	13,935		8,650	
Standard deviation	2,629		2,170	

[1]Flow measurement occurred on September 17, 1999

Table A12. Selected flow measurements on Detroit River near the gap in Livingstone Channel, Ontario, Canada

[ft^3/s, cubic feet per second]

Measurement date	Flow at CS-141 (ft^3/s)	Number of Transects	Flow at CS-163 (ft^3/s)	Number of transects
June 25, 1996	22,800	3	44,200	4
July 23, 1996	21,900	4	42,900	3
May 2, 1997	23,200	3	31,500	4
May 21, 1997	26,700	2	43,400	2
June 25, 1997	21,900	3	42,200	2
July 23, 1997	22,600	2	41,000	3
August 19, 1997	24,500	3	45,300	2
September 24, 1997	26,600	2	41,600	3
October 23, 1997	25,900	2	43,900	3
August 4, 1998	22,200	2	41,400	2
September 9, 1998	23,800	1	47,600	2
October 5, 1998	21,900	2	44,000	2
May 28, 1999	20,200	3	35,500	2
July 7, 1999	19,200	2	42,100	2
September 16, 1999	18,600	2	[1]46,700	2
October 7, 1999	21,500	2	44,000	2
November 18, 1999	23,100	2	34,200	2
Average	22,888		41,555	
Standard deviation	2,329		4,355	

[1]Flow measurement occurred on September 17, 1999

Table A13. Selected flow measurements on Detroit River near Sugar Island, Michigan

[ft^3/s, cubic feet per second]

Measurement date	Flow at CS-161 (ft^3/s)	Number of Transects	Flow at CS-162 (ft^3/s)	Number of transects
June 25, 1996	36,600	2	13,800	4
July 23, 1996	29,900	2	11,700	3
May 2, 1997	30,700	2	11,300	3
May 21, 1997	40,000	3	14,400	3
June 25, 1997	36,200	3	12,400	2
July 23, 1997	30,400	2	11,900	2
August 19, 1997	31,100	3	13,600	2
September 24, 1997	29,800	3	12,000	3
October 23, 1997	31,200	2	13,200	3
August 4, 1998	28,200	2	11,500	2
September 9, 1998	27,300	2	12,500	2
October 5, 1998	26,200	3	10,900	2
July 7, 1999	21,000	2	8,480	4
September 17, 1999	19,700	2	9,600	2
October 7, 1999	19,000	2	9,700	3
November 18, 1999	22,100	2	8,500	2
Average	28,890		11,595	
Standard deviation	5,943		1,740	

APPENDIX B.
Example of Matlab Function Developed to Simultaneously Estimate Reach Parameters

Example of MATLAB function developed to simultaneously estimate reach parameters

```
% Detroit River flow proportions
% Arguments: b is a vector of parameters, and X is a vector of flow in the reach
N = length(X);
yhat = zeros(N,1);
%
% American Grassy Island near cross section CS-100
for i = 1:N/2,
    yhat(i) = b(1);
end
% Fighting Island Channel near cross section CS-101
for i = N/2+1:N,
    yhat(i) = b(2)+b(3)*X(i);
end
```

APPENDIX C.

Listing of Matlab Code Used in the Monte Carlo Analysis of Flow Magnitudes and Uncertainties

Listing of Matlab[2] Code Used in the Monte Carlo Analysis of Flow Magnitudes and Uncertainties

```
% to compute expected values and uncertainties of proportions and flows through St. Clair and Detroit Rivers
% on the basis of specified flows, by D.J. Holtschlag, USGS May 2001
%
% Simulation parameters
function SimSCD4
Nsim = 1000000;
alpha = 0.05;
%
% Specified flows at boundaries
for Qsc = 170000:10000:230000
% Qsc = 180000;  % Streamflow at headwater of St. Clair River
Qbl =    489;  % Streamflow at mouth of Black River on St. Clair River
Qpi =    119;  % Streamflow at mouth of Pine River on St. Clair River
Qbe =    478;  % Streamflow at mouth of Belle River on St. Clair River
Qsy =   1861;  % Streamflow at the mouth of Sydenham River on Lake St. Clair
Qth =   4857;  % Streamflow at the mouth of Thames River on Lake St. Clair
Qcl =    928;  % Streamflow at the mouth of Clinton River
Qol =    626;  % Overlake contribution from Lake St. Clair
Qro =    312;  % Streamflow from River Rouge
%
fprintf(1,'Analysis of Flow Distribution in Branches of St. Clair and Detroit Rivers \n');
fprintf(1,'Input flows: Mouth of St. Clair River: Qsc %8.0f \n',Qsc);
fprintf(1,'             Mouth of Black River:     Qbl %8.0f \n',Qbl);
fprintf(1,'             Mouth of Pine River:      Qpi %8.0f \n',Qpi);
fprintf(1,'             Mouth of Belle River:     Qbe %8.0f \n',Qbe);
fprintf(1,'             Sydenham River at gauge   Qsy %8.0f \n',Qsy);
fprintf(1,'             Thames River at gauge     Qst %8.0f \n',Qth);
fprintf(1,'             Clinton River at gauge    Qcl %8.0f \n',Qcl);
fprintf(1,'             Net over Lake St. Clair   Qol %8.0f \n',Qol);
fprintf(1,'             River Rouge at gauge      Qro %8.0f \n',Qro);
fprintf(1,' \n');
%
% * * * ST. CLAIR RIVER * * * ST. CLAIR RIVER * * * ST. CLAIR RIVER * * * ST. CLAIR RIVER * * *
%
% STAG ISLAND REACH
%
% Stag Island East Branch
PExpd208 = 0.25185+3.9158e-7*(Qsc+Qbl);      % Proportion of flow expected in CS-208
n208 = 17; m208 =  2;                        % Number of measurements and parameters in regression equation
SEEsti208 = 0.0063394;                       % Standard error of the estimate
xbar = 203980; xstd = 17009;                 % Mean and standard deviation of flow measured in reach
SEPred208 = SEEsti208 * sqrt(1+1/n208+(Qsc+Qbl-xbar)^2/( (n208-m208)*xstd^2));    % Standard error of
prediction
PSamp208 = PExpd208+trnd(n208-m208,Nsim,1).*SEPred208;        % Sample of proportions from T dist
qSamp208 = PSamp208 * (Qsc+Qbl);             % Sample of corresponding flows
%
% Stag Island West Branch
PSamp210 = 1 - PSamp208;                     % Continuity constraint estimate of flow proportions at CS-210
qSamp210 = PSamp210 * (Qsc+Qbl);             % Sample of corresponding flows
```

[2]MATLAB is a proprietary computer language and software program for technical computation, programming, and visualization by the Math Works, Inc.

```
% MATLAB function SimSCD4
%
fprintf(1,'            Cross    Expect  L95CI   U95CI   Expect   L95CI   U95CI   Expect  L95CI   U95CI  StdDev
StdDev   Variance  \n');
fprintf(1,'REACH           Section   Propor   Propor   Propor   FxdFlow  FxdFlow  FxdFlow  RndFlow  RndFlow
RndFlow  FxdFlow  RndFlow  RndQ/FxdQ  \n');
fprintf(1,'Stag Island       CS-208   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f  \n',...
                         mean(PSamp208),prctile(PSamp208,[2.5, 97.5]),...
                         mean(qSamp208),prctile(qSamp208,[2.5, 97.5]));
fprintf(1,'                  CS-210   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f  \n',...
                         mean(PSamp210),prctile(PSamp210,[2.5, 97.5]),...
                         mean(qSamp210),prctile(qSamp210,[2.5, 97.5]));
%
clear PSamp208 qSamp208 PSamp210 qSamp210
%
% FAWN ISLAND REACH
%
% Fawn Island West Branch
PExpd216 = 0.87708-2.8634e-7*(Qsc+Qbl+Qpi+Qbe);
n216 = 19; m216 = 2;
SEEsti216 = 0.0047136;
xbar = 200475; xstd = 18444;
SEPred216 = SEEsti216 * sqrt(1+1/n216+(Qsc+Qbl+Qpi+Qbe-xbar)^2/( (n216-m216)*xstd^2));
PSamp216 = PExpd216+trnd(n216-m216,Nsim,1).*SEPred216;
qSamp216 = PSamp216 * (Qsc+Qbl+Qpi+Qbe);
%
% Fawn Island East Branch
PSamp218 = 1 - PSamp216;
qSamp218 = PSamp218 * (Qsc+Qbl+Qpi+Qbe);
%
fprintf(1,'Fawn Island       CS-216   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f  \n',...
                         mean(PSamp216),prctile(PSamp216,[2.5, 97.5]),...
                         mean(qSamp216),prctile(qSamp216,[2.5, 97.5]));
fprintf(1,'                  CS-218   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f  \n',...
                         mean(PSamp218),prctile(PSamp218,[2.5, 97.5]),...
                         mean(qSamp218),prctile(qSamp218,[2.5, 97.5]));
%
clear PSamp216 qSamp216 PSamp218 qSamp218
%
% WALPOLE ISLAND REACH
%
% Chenal Ecarte Branch
PExpd222 = 2.1066e-7*(Qsc+Qbl+Qpi+Qbe);
n222 = 17;  m222 = 1;
SEEsti222 = 0.0032261;
xbar = 203980;  xstd = 14829;
SEPred222 = SEEsti222 * sqrt(1+1/n222+(Qsc+Qbl+Qpi+Qbe-xbar)^2/( (n222-m222)*xstd^2));
PSamp222 = PExpd222+trnd(n222-m222,Nsim,1)*SEPred222;
qSamp222 = PSamp222 * (Qsc+Qbl+Qpi+Qbe);          % Flow uncertainty associated with the uncertainty of
prop
%
% Mouth of St. Clair River Branch
PSamp230 = 1-PSamp222;
qSamp230 = PSamp230 * (Qsc+Qbl+Qpi+Qbe);
%
fprintf(1,'Chenal Ecarte      CS-222   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f \n',...
                         mean(PSamp222),prctile(PSamp222,[2.5, 97.5]),...
                         mean(qSamp222),prctile(qSamp222,[2.5, 97.5]));
```

```
% MATLAB function SimSCD4
fprintf(1,'St. Clair River    CS-230   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f \n',...
                        mean(PSamp230),prctile(PSamp230,[2.5, 97.5]),...
                        mean(qSamp230),prctile(qSamp230,[2.5, 97.5]));
%
clear PSamp222 qSamp222 PSamp230
%
% HARSENS-DICKINSON ISLAND REACH
%
% South Channel Branch
PExpd232  = 0.54859 - 4.6909e-07 .* qSamp230;
n232      = 16; m232 = 2;
SEEsti232 = 0.011858;
xbar      = 191419; xstd = 14682;
SEPred232 = SEEsti232 .* sqrt(1 + 1/n232 +  (qSamp230 - xbar).^2 ./ (( n232-m232)*xstd^2));
%
% North Channel Branch
PExpd240  = 0.24258 + 5.9493e-07 .* qSamp230;
n240      = 16; m240 = 2;
SEEsti240 = 0.008936;
SEPred240 =  SEEsti240 .* sqrt(1 + 1/n240 + (qSamp230 - xbar).^2 ./ (( n240-m240)*xstd^2));
%
% Middle Channel Branch
PExpd242 = 1 - PExpd232 - PExpd240;
n242 = 16; m240 = 2;
SEEsti242 = 0.010047;
%
% Generate random numbers with proper covariance structure
CorP1P2 = [1 -0.529978; -0.529978 1];  % Based on analysis of data;
randvec = trnd(n240-m240,Nsim,2);
for i=1:Nsim,
   CovP1P2 = diag([SEPred232(i) SEPred240(i)]) * CorP1P2 * diag([SEPred232(i)  SEPred240(i)]);
   randvec(i,:)  = randvec(i,:)*sqrtm(CovP1P2);  % This randvec is the same size but the right variance.
end
%
PSamp232 = PExpd232 + randvec(:,1); % Realization of proportions reflecting the uncertainty of the propor-
tions.
qSamp232 = PSamp232  * mean(qSamp230);   % Realization of flows reflecting the uncertainty of the propor-
tions.
QSamp232 = PSamp232 .*    qSamp230;     % Realization of flows reflecting the uncertainty of the proportions
and upstream flows.

PSamp240 = PExpd240  + randvec(:,2);  % Realization of proportions reflecting the uncertainty of the propor-
tions.
qSamp240 = PSamp240  * mean(qSamp230);   % Realization of flows reflecting the uncertainty of the propor-
tions.
QSamp240 = PSamp240 .*    qSamp230;    % Realization of flows reflecting the uncertainty of the proportions
and upstream flows.

PSamp242  = 1 - PSamp232 - PSamp240;       % Implications of continuity restriction on proportions.
qSamp242  = PSamp242  * mean(qSamp230);
QSamp242  = PSamp242 .*     qSamp230;
%
%
%
fprintf(1,'South Channel     CS-232   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f %8.0f %8.0f %8.0f %9.1f %9.1f
%8.4f \n',...
                        mean( PSamp232),prctile( PSamp232,[2.5, 97.5]),...
                        mean( qSamp232),prctile( qSamp232,[2.5, 97.5]),...
                        mean( QSamp232),prctile( QSamp232,[2.5, 97.5]),...
                        std(qSamp232),std(QSamp232),var(QSamp232)/var(qSamp232));
```

```
% MATLAB function SimSCD4
%
fprintf(1,'North Channel CS-240 %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f %8.0f %8.0f %8.0f %9.1f %9.1f
%8.4f \n',...
                         mean( PSamp240),prctile( PSamp240,[2.5, 97.5]),...
                         mean( qSamp240),prctile( qSamp240,[2.5, 97.5]),...
                         mean( QSamp240),prctile( QSamp240,[2.5, 97.5]),...
                         std(qSamp240),std(QSamp240),var(QSamp240)/var(qSamp240));

%
fprintf(1,'Middle Channel    CS-242   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f %8.0f %8.0f %8.0f %9.1f %9.1f
%8.4f \n',...
                         mean( PSamp242),prctile( PSamp242,[2.5, 97.5]),...
                         mean( qSamp242),prctile( qSamp242,[2.5, 97.5]),...
                         mean( QSamp242),prctile( QSamp242,[2.5, 97.5]),...
                         std(qSamp242),std(QSamp242),var(QSamp242)/var(qSamp242));
%
clear PSamp232 PSamp240 qSamp240 QSamp240 PSamp242 qSamp242 QSamp242
%
% BASSETT-SEAWAY ISLAND REACH
%
% St. Clair Cutoff Branch
PExpd236  = 0.54025;
n236      = 17; m236 = 1;
SEEsti236 = 0.022840;
xbar      = 86667; xstd = 5907;
SEPred236 = SEEsti236;
%
% St. Clair Flats Branch
PExpd238 = 0.37914;
n238 = 17; m238 = 1;
SEEsti238 = 0.027820;
SEPred238 = SEEsti238;
%
% Bassett Channel Branch
PExpd234 = 1 - PExpd236 - PExpd238;
%
CorP1P2 = [   1   -0.94082; -0.94082   1];  % Based on an analysis of data
randvec = trnd(n236-m236,Nsim,2);
CovP1P2 = diag([SEPred236 SEPred238]) * CorP1P2 * diag([SEPred236 SEPred238]);
randvec = randvec*sqrtm(CovP1P2);
%
PSamp236 = PExpd236 + randvec(:,1);    % Realization of proportions based on the expected value and uncer-
tainty of the proportions
qSamp236 = PSamp236 * mean(QSamp232);   % Realization of flows reflecting the uncertainty of the propor-
tions
QSamp236 = PSamp236 .*   QSamp232;    % Realization of flows reflecting the uncertainty of the proportions
and the upstream flows

PSamp238  = PExpd238  + randvec(:,2);
qSamp238  = PSamp238  * mean(QSamp232);
QSamp238  = PSamp238 .*      QSamp232;          %

PSamp234  = 1 - PSamp236 - PSamp238;
qSamp234  = PSamp234  * mean(QSamp232);
QSamp234  = PSamp234 .*      QSamp232;
%
%
```

```
% MATLAB function SimSCD4
fprintf(1,'Bassett Channel CS-234 %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f %8.0f %8.0f %8.0f %9.1f %9.1f %8.4f
\n',...
                    mean( PSamp234),prctile( PSamp234,[2.5, 97.5]),...
                    mean( qSamp234),prctile( qSamp234,[2.5, 97.5]),...
                    mean( QSamp234),prctile( QSamp234,[2.5, 97.5]),...
                    std(qSamp234),std(QSamp234),var(QSamp234)/var(qSamp234) );
%
fprintf(1,'St. Clair Cutoff CS-236  %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f %8.0f %8.0f %8.0f %9.1f %9.1f %8.4f
\n',...
                    mean( PSamp236),prctile( PSamp236,[2.5, 97.5]),...
                    mean( qSamp236),prctile( qSamp236,[2.5, 97.5]),...
                    mean( QSamp236),prctile( QSamp236,[2.5, 97.5]),...
                    std(qSamp236),std(QSamp236),var(QSamp236)/var(qSamp236));

%
fprintf(1,'St. Clair Flats CS-238   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f %8.0f %8.0f %8.0f %9.1f %9.1f %8.4f
\n',...
                    mean( PSamp238),prctile( PSamp238,[2.5, 97.5]),...
                    mean( qSamp238),prctile( qSamp238,[2.5, 97.5]),...
                    mean( QSamp238),prctile( QSamp238,[2.5, 97.5]),...
                    std(qSamp238),std(QSamp238),var(QSamp238)/var(qSamp238));
%
clear PSamp236 qSamp236 QSamp236 PSamp238 qSamp238 QSamp238 PSamp234 qSamp234 QSamp234 randvec
QSamp232 qSamp232
%
% * * * DETROIT RIVER  * * * DETROIT RIVER * * * DETROIT RIVER * * * DETROIT RIVER * * *
%
% PECHE ISLAND REACH
%
% Peche Island North Branch
PExpd003   = 0.73503;
n003    = 19; m003 = 1;
SEEsti003  = 0.009451;                          % Standard error
SEPred003  = SEEsti003;
PSamp003   = PExpd003+trnd(n003-m003,Nsim,1)*SEPred003;
qSamp003   = PSamp003 * (Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol);
% Peche Island South Branch
PSamp008   = 1-PSamp003;
qSamp008   = PSamp008 * (Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol);
%
fprintf(1,'Peche Island       CS-3     %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f  \n',...
                    mean(PSamp003),prctile(PSamp003,[2.5, 97.5]),...
                    mean(qSamp003),prctile(qSamp003,[2.5, 97.5]));
%
fprintf(1,'                   CS-8     %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f  \n',...
                    mean(PSamp008),prctile(PSamp008,[2.5, 97.5]),...
                    mean(qSamp008),prctile(qSamp008,[2.5, 97.5]));
%
clear PSamp003 qSamp003 PSamp008 qSamp008
%
% BELLE ISLE REACH
%
% Scott Middle Ground Branch
PExpd015  = 0.21262+5.0612e-7*(Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol);        % Proportion of flow in cs15
n015      = 19; m015 = 2;                                        % Effective n = N-p;
SEEsti015 = 0.007104;
xbar      = 214900; xstd = 16069;
SEPred015 = SEEsti015 * sqrt(1 + 1/n015 + (Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol-xbar)^2/((n015-% MATLAB
```

```
function SimSCD4
m015)*xstd^2));
PSamp015  = PExpd015+trnd(n015-m015,Nsim,1)*SEPred015;
qSamp015  = PSamp015  * (Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol);         % Expected flow thru cs15
% Fleming Channel Branch
PSamp029  = 1 - PSamp015;                                           % Expected flow thru cs15
qSamp029  = PSamp029  * (Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol);
%
fprintf(1,'Belle Isle         CS-15    %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f  \n',...
                        mean(PSamp015),prctile(PSamp015,[2.5, 97.5]),...
                        mean(qSamp015),prctile(qSamp015,[2.5, 97.5]));
%
fprintf(1,'                   CS-29    %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f  \n',...
                        mean(PSamp029),prctile(PSamp029,[2.5, 97.5]),...
                        mean(qSamp029),prctile(qSamp029,[2.5, 97.5]));
%
clear PSamp015 qSamp015 PSamp029 qSamp029
%
% GRASSY-FIGHTING ISLAND REACH
%
% American Grassy Island Branch
PExpd100  = 0.26130;
n100      = 18; m100 = 1;
xbar      = 212320; xstd = 23002;
SEEsti100 = 0.007224;
SEPred100 = SEEsti100;
%
% Fighting Island Branch
PExpd101  = 0.55823-1.8722e-7*(Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol+Qro);
n101      = 18; m101 = 2;
SEEsti101 = 0.005641;
SEPred101 = SEEsti101 * sqrt(1 + 1/n101 + (Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol-xbar)^2/(( n101-
m101)*xstd^2));
%
% Canadian Grassy Island Branch
PExpd102 = 1 - PExpd100 -  PExpd101;
%
CorP1P2 = [ 1  -.76025 ; -.76025 1];
randvec = trnd(n101-m101,Nsim,2);
CovP1P2 = diag([SEPred100 SEPred101]) * CorP1P2 * diag([SEPred100 SEPred101]);
randvec = randvec*sqrtm(CovP1P2);
%
PSamp100  = PExpd100  + randvec(:,1);       % Just added a new random component to PExpd232
qSamp100  = PSamp100 .* (Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol+Qro);         % This is a new flow that reflects
uncertainty inboth p232 and p230 keeping the
PSamp101  = PExpd101  + randvec(:,2);
qSamp101  = PSamp101 .* (Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol+Qro);          %
PSamp102  = 1 - PSamp100 - PSamp101;
qSamp102  = PSamp102 .* (Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol+Qro);
%
%
fprintf(1,'American Grassy    CS-100   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f  \n',...
                        mean(PSamp100),prctile(PSamp100,[2.5, 97.5]),...
                        mean(qSamp100),prctile(qSamp100,[2.5, 97.5]));
%
fprintf(1,'Fighting Island    CS-101   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f  \n',...
                        mean(PSamp101),prctile(PSamp101,[2.5, 97.5]),...
```

```
                        mean(qSamp101),prctile(qSamp101,[2.5, 97.5]));
%
fprintf(1,'Canadian Grassy    CS-102   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f  \n',...
                        mean(PSamp102),prctile(PSamp102,[2.5, 97.5]),...
                        mean(qSamp102),prctile(qSamp102,[2.5, 97.5]));
%
clear PSamp100 qSamp100 PSamp101 qSamp101 PSamp102 qSamp102
%
% GROSSE-STONY-DIKE REACH
%
% Trenton Channel Branch
PExpd120  = 0.22156;
n120      = 17; m120 = 1;
xbar      = 210831;  xstd = 18839;
SEEsti120 = 0.0099257;
SEPred120 = SEEsti120;
%
% Grosse-Stony Branch
PExpd121  = -0.11248+9.0125e-7*(Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol+Qro);
n121      = 17; m121 = 2;
SEEsti121 = 0.010892;
SEPred121 = SEEsti121 * sqrt(1 + 1/n121 + (Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol+Qro-xbar)^2/((n121-
m121)*xstd^2));
%
% Upstream Livingston Branch
PExpd122  = 0.34521 -4.4833e-7*(Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol+Qro);
n122 = 17; m122 = 2;
SEEsti122 = 0.0088991;
SEPred122 = SEEsti122 * sqrt(1 + 1/n122 + (Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol+Qro-xbar)^2/((n122-
m122)*xstd^2));
%
% Upstream Amherstburg Branch
PExpd123 = 1 - PExpd120 - PExpd121 - PExpd122;
%
CorP1P2P3 = [1  .39854  -.75693; .39854    1    -.66573;  -.75693  -.66573   1];
CovP1P2P3 = diag([SEPred120 SEPred121 SEPred122]) * CorP1P2P3 * diag([SEPred120 SEPred121
SEPred122]);
randvec   = trnd(n122-m122,Nsim,3)*sqrtm(CovP1P2P3);
%
PSamp120  = PExpd120  + randvec(:,1);
qSamp120  = PSamp120 .* (Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol+Qro);
PSamp121  = PExpd121  + randvec(:,2);
qSamp121  = PSamp121 .* (Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol+Qro);
PSamp122  = PExpd122  + randvec(:,3);
qSamp122  = PSamp122 .* (Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol+Qro);
PSamp123  = 1 - PSamp120 - PSamp121 - PSamp122;
qSamp123  = PSamp123 .* (Qsc+Qbl+Qpi+Qbe+Qsy+Qth+Qcl+Qol+Qro);
%
fprintf(1,'Trenton Channel    CS-120   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f  \n',...
                        mean(PSamp120),prctile(PSamp120,[2.5, 97.5]),...
                        mean(qSamp120),prctile(qSamp120,[2.5, 97.5]));
%
fprintf(1,'Grosse-Stony       CS-121   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f  \n',...
                        mean(PSamp121),prctile(PSamp121,[2.5, 97.5]),...
                        mean(qSamp121),prctile(qSamp121,[2.5, 97.5]));
%
fprintf(1,'US Livingstone     CS-122   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f  \n',...
```

```
% MATLAB function SimSCD4
                        mean(PSamp122),prctile(PSamp122,[2.5, 97.5]),...
                        mean(qSamp122),prctile(qSamp122,[2.5, 97.5]));
%
fprintf(1,'US Amherstburg     CS-123   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f  \n',...
                        mean(PSamp123),prctile(PSamp123,[2.5, 97.5]),...
                        mean(qSamp123),prctile(qSamp123,[2.5, 97.5]));
%
clear PSamp120 qSamp120 PSamp121 PSamp122 PSamp123 randvec SEPred232 SEPred232 SEPred240
clear PExpd232 PExpd240 PExpd242
%
% BOIS BLANC ISLAND REACH
%
% Amherstburg Gap Branch
PExpd143  = 0.24095;
n143      = 16; m143 = 1;
SEEsti143 = 0.018619;
SEPred143 = SEEsti143;
PSamp143  = PExpd143+trnd(n143-m143,Nsim,1)*SEPred143;
qSamp143  = PSamp143   * mean(qSamp123);
QSamp143  = PSamp143  .*      qSamp123;
%
% Lower Amherstburg Branch
PExpd165  = 1 - PExpd143;
PSamp165  = 1 - PSamp143;
qSamp165  = PSamp165   * mean(qSamp123);
QSamp165  = PSamp165  .*      qSamp123 ;
%
fprintf(1,'Amherstburg Gap    CS-143   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f %8.0f %8.0f %8.0f %9.1f
%9.1f %8.4f \n',...
                        mean(PSamp143),prctile(PSamp143,[2.5, 97.5]),...
                        mean(qSamp143),prctile(qSamp143,[2.5, 97.5]),...
                        mean(QSamp143),prctile(QSamp143,[2.5, 97.5]),...
                        std( qSamp143),std(QSamp143),var(QSamp143)/var(qSamp143));
%
fprintf(1,'DS Amherstburg     CS-165   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f %8.0f %8.0f %8.0f %9.1f
%9.1f %8.4f \n',...
                        mean(PSamp165),prctile(PSamp165,[2.5, 97.5]),...
                        mean(qSamp165),prctile(qSamp165,[2.5, 97.5]),...
                        mean(QSamp165),prctile(QSamp165,[2.5, 97.5]),...
                        std( qSamp165),std(QSamp165),var(QSamp165)/var(qSamp165) );
%
clear PSamp143 qSamp143 PSamp165 qSamp165 QSamp165
%
%
% BOIS BLANC-LIVINGSTONE GAP REACH
%
% Livingstone Gap East Branch
PExpd142  = 0.59822;
n142 = 17; m142 = 1;
SEEsti142 = 0.089521;
SEPred142 = SEEsti142;
PSamp142 = PExpd142+trnd(n142-m142,Nsim,1)*SEPred142;
qSamp142 = PSamp142  * mean(QSamp143);                % Expected flow thru cs142
QSamp142 = PSamp142 .*      QSamp143;
%
% Bois Blanc-Dike Branch
```

```
% MATLAB function SimSCD4
PExpd164 = 1 - PExpd142;
PSamp164 = 1 - PSamp142;
qSamp164 = PSamp164  * mean(QSamp143);              % Expected flow thru cs142
QSamp164 = PSamp164 .*      QSamp143;
%
fprintf(1,'Livingstone Gap E  CS-142   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f %8.0f %8.0f %8.0f %9.1f
%9.1f %8.4f \n',...
                        mean(PSamp142),prctile(PSamp142,[2.5, 97.5]),...
                        mean(qSamp142),prctile(qSamp142,[2.5, 97.5]),...
                        mean(QSamp142),prctile(QSamp142,[2.5, 97.5]),...
                        std( qSamp142),std(QSamp142),var(QSamp142)/var(qSamp142) );
%
%8.5f         %8.5f         %8.5f        \n',mean(  r142),prctile(  r142,[2.5, 97.5]));
fprintf(1,'Bois Blanc-Dike    CS-164   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f %8.0f %8.0f %8.0f %9.1f
%9.1f %8.4f \n',...
                        mean(PSamp164),prctile(PSamp164,[2.5, 97.5]),...
                        mean(qSamp164),prctile(qSamp164,[2.5, 97.5]),...
                        mean(QSamp164),prctile(QSamp164,[2.5, 97.5]),...
                        std( qSamp164),std(QSamp164),var(QSamp164)/var(qSamp164) );
%
clear PSamp164 qSamp164 QSamp164
%
% LIVINGSTONE GAP REACH
%
% Livingstone Gap West Branch
PExpd141  = 0.35606;
n141      = 18; m141 = 1;
SEEsti141 = 0.035504;
SEPred141 = SEEsti141;
PSamp141  = PExpd141+trnd(n141-m141,Nsim,1)*SEPred141;

qSamp141  = PSamp141  * mean(qSamp122 + QSamp142);
QSamp141  = PSamp141 .*     (qSamp122 + QSamp142);
%
% Lower Livingstone Branch
PExpd163  = 1 - PExpd141;
PSamp163  = 1 - PSamp141;
qSamp163  = PSamp163  * mean(qSamp122 + QSamp142);
QSamp163  = PSamp163 .*     (qSamp122 + QSamp142);

QSamp163  = qSamp122 + QSamp142 - QSamp141;
%
fprintf(1,'Livingstone Gap W  CS-141   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f %8.0f %8.0f %8.0f %9.1f
%9.1f %8.4f \n',...
                        mean(PSamp141),prctile(PSamp141,[2.5, 97.5]),...
                        mean(qSamp141),prctile(qSamp141,[2.5, 97.5]),...
                        mean(QSamp141),prctile(QSamp141,[2.5, 97.5]),...
                        std( qSamp141),std(QSamp141),var(QSamp141)/var(qSamp141) );
%
fprintf(1,'DS Livingstone     CS-163   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f %8.0f %8.0f %8.0f %9.1f %9.1f
%8.4f \n',...
                        mean(PSamp163),prctile(PSamp163,[2.5, 97.5]),...
                        mean(qSamp163),prctile(qSamp163,[2.5, 97.5]),...
                        mean(QSamp163),prctile(QSamp163,[2.5, 97.5]),...
                        std( qSamp163),std(QSamp163),var(QSamp163)/var(qSamp163) );
%
```

```
% MATLAB function SimSCD4
%
clear PSamp141 qSamp141 PSamp163 qSamp163 QSamp163
%
% SUGAR ISLAND REACH
%
% West Sugar Island Branch
PExpd161  = 0.63465+1.8888e-6 .* (qSamp121+QSamp141);
n161      = 17; m161 = 2;
xbar      = 40485; xstd = 7526;
SEEsti161 = 0.017062;
SEPred161 = SEEsti161 * sqrt(1 + 1/n161 + (qSamp121+QSamp141-xbar).^2 ./ ( (n161-m161)*xstd^2));
PSamp161  = PExpd161+trnd(n161-m161,Nsim,1).*SEPred161;
qSamp161  = PSamp161  * mean(qSamp121 + QSamp141);
QSamp161  = PSamp161 .*     (qSamp121 + QSamp141);
%
% East Sugar Island Branch
PSamp162  = 1 - PSamp161;
qSamp162  = PSamp162  * mean(qSamp121 + QSamp141);
QSamp162  = PSamp162 .*     (qSamp121 + QSamp141);

fprintf(1,'W Sugar Island     CS-161   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f %8.0f %8.0f %8.0f %9.1f %9.1f
%8.4f \n',...
                         mean(PSamp161),prctile(PSamp161,[2.5, 97.5]),...
                         mean(qSamp161),prctile(qSamp161,[2.5, 97.5]),...
                         mean(QSamp161),prctile(QSamp161,[2.5, 97.5]),...
                         std(qSamp161),std(QSamp161),var(QSamp161)/var(qSamp161) );
%
fprintf(1,'E Sugar Island     CS-162   %8.5f %8.5f %8.5f %8.0f %8.0f %8.0f %8.0f %8.0f %8.0f %9.1f %9.1f
%8.4f \n',...
                         mean(PSamp162),prctile(PSamp162,[2.5, 97.5]),...
                         mean(qSamp162),prctile(qSamp162,[2.5, 97.5]),...
                         mean(QSamp162),prctile(QSamp162,[2.5, 97.5]),...
                         std(qSamp162),std(QSamp162),var(QSamp162)/var(qSamp162) );
%
clear PSamp161 qSamp162 QSamp161 PSamp162 qSamp162 QSamp162
%
end
```